MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

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Dr. J. H. JEANS, PRESIDENT, in the Chair.

Ethel Frances Butwell Bellamy, Assistant at the University Observatory, Oxford, 28 Polstead Road, Oxford;
George Frederick Morrell, Avenue House, Holly Park, Crouch Hill, London, N. 4; and
Winthrop Robins Wright, Professor of Physics, Swarthmore College, Swarthmore, Pa., U.S.A., and Cavendish Laboratory, Cambridge,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Philip Arthur Curry, B.A., A.Inst.P., Director, Helwan Observatory, Egypt (proposed by H. Knox-Shaw).

One hundred and twenty-five presents were announced as having been received since the last Ordinary Meeting, including, amongst others:—

R. M. Abraham: Surveying Instruments.
C. V. L. Charlier: The Motion and the Distribution of the Stars (University of California Memoirs, vol. 7).
Córdoba Observatory: Astrographic Catalogue, vol. 26, zone $-24°$.

Dearborn Observatory: Publications, vol. 2, Measures of Double Stars.

J. G. Hagen: Preparatory Catalogue for a Durchmusterung of Nebulæ. Second Instalment, zones 60° to 80° N.P.D.

Leander McCormick Observatory: Publications, vol. 2, pts. 5 and 7.

R. A. Proctor: Half-hours with the Telescope. Revised and brought up to date by W. H. Steavenson.

San Fernando Observatory: Astrographic Chart, zone −5° (10 charts).

F. J. W. Schroeter: Haandbog i Kronologie, vol. 2.

The Orbit of Neptune's Satellite and the Pole of Neptune's Equator.

By W. S. Eichelberger and Arthur Newton.

(*Communicated officially by the Superintendent, U.S. Naval Observatory.*)

Several years after the death of Professor Simon Newcomb, the U.S. *Nautical Almanac* Office, in response to a suggestion from its Director, received from the family of Professor Newcomb his uncompleted work on the orbit of the satellite of Neptune.

The following statement concerning the work was left by Professor Newcomb :—

"The most complete discussion of the orbit of Neptune's satellite that has been published up to the present time is that of Hermann Struve, found in his memoir, "Beobachtungen des Neptunstrabanten am 30-zölligen Pulkowaer Refractor," *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg*, vii^e^ serie, tome xlii., No. 4. In this memoir are discussed all the observations made from the time of the discovery of the satellite by Lassell in 1846 up to 1892.

"During the period which has since elapsed a mass of observations has accumulated much greater than that made use of by Struve. Of these, several special series have been worked up by Professors See, Brown, and others, but none of these includes a complete discussion of all the material.

"Added interest is given to the subject by the application of the photographic method which has been made at several observatories, notably those of Mount Hamilton and Greenwich. These discussions may be supposed almost entirely free from the systematic personal errors to which eye observations, especially of distance, are liable. The magnitude of these errors is shown in the diversity among the values of the mass of Neptune derived from the work of different observers.

"In the present work no attempt has been made to rediscuss those observations from which the resulting elements have already been sufficiently derived. This seems to be the case with all the observations made use of by Professor Struve. It has, however, been deemed best to rediscuss all the observations (1892–1908) made since the publication of his memoir, using the few published discussions which exist as checks against accidental errors in computation.

"*Provisional Theory.*

"The adopted provisional elements of the satellite are derived from the work of Struve.* We follow the usual notation:

N=the right ascension of the ascending node of the orbit on the plane of the Earth's equator,
I=the inclination of the orbit to the same plane,
$u_\oplus$=the mean angular distance of the satellite in its orbit from the ascending node of the orbit on the plane of the Earth's equator,
a=the semi-major axis of the orbit.

"The adopted numerical values are [1889–1908].

$u_\oplus = 234^\circ{\cdot}42$ for 1890 Jan. 0, G.M.N.,
$n = 61^\circ{\cdot}25748$,
$N = 185^\circ{\cdot}15 + 0^\circ{\cdot}148\ (T-1890{\cdot}0)$,
$I = 119^\circ{\cdot}35 - 0^\circ{\cdot}165\ (T-1890{\cdot}0)$,
$a = 16''{\cdot}30$, for mean distance [1·47814],
$e = 0$.

Here n is the daily motion of $u_\oplus$, not the sidereal motion of the satellite."

Comparison of Observation with Provisional Theory.

The formulæ used for computing the values of p and s, the position angle and distance, are, using the notation of Struve,

$$\begin{aligned}
s \sin (p-\mathrm{P}) &= r \sin (u_\oplus - \mathrm{U}),\\
s \cos (p-\mathrm{P}) &= r \sin \mathrm{B} \cos (u_\oplus - \mathrm{U}),\\
\cos \mathrm{B} \sin \mathrm{U} &= \cos\delta \sin(\alpha-\mathrm{N}) \cos \mathrm{I} + \sin\delta \sin \mathrm{I},\\
\cos \mathrm{B} \cos \mathrm{U} &= \cos\delta \cos(\alpha-\mathrm{N}),\\
\sin \mathrm{B} &= \cos\delta \sin(\alpha-\mathrm{N}) \sin \mathrm{I} - \sin\delta \cos \mathrm{I},\\
\cos \mathrm{B} \sin \mathrm{P} &= -\cos(\alpha-\mathrm{N}) \sin \mathrm{I},\\
\cos \mathrm{B} \cos \mathrm{P} &= \sin\delta \sin(\alpha-\mathrm{N}) \sin \mathrm{I} + \cos\delta \cos \mathrm{I},
\end{aligned}$$

where

$$r = \frac{a[1{\cdot}47814]}{\rho} = \frac{16''{\cdot}30[1{\cdot}47814]}{\rho} = \frac{[2{\cdot}69033]}{\rho},$$

and ρ, α, δ are the geocentric co-ordinates of Neptune.

Under the direction of Professor Newcomb the position angle and distance had been computed from the provisional elements for the time of each of 1004 observations of Neptune's satellite made from 1889 to 1908, and much work had been done on the computation of the differential coefficients of the equations of condition for determining the corrections to the provisional elements. As there was uncertainty about just what portions of this work had been checked, a duplicate computation covering the entire period was made in the *Nautical Almanac* Office, and the list of observations to be discussed was extended from 1908 to 1923.

* The values of a and e were adopted by Newcomb, and are not the result of Struve's discussion.

Equations of Condition and Normal Equations.

Each complete observation gave two equations of condition between the corrections to the assumed elements of the form :

$$\begin{aligned} s \sin dp = {} & r \sin \tau \,.\, \sin du_{\oplus} \\ & + (r \sin \tau \cos \mathrm{I} + r \cos \tau \cos u_{\oplus} \sin \mathrm{I}) \,.\, \sin d\mathrm{N} \\ & - r \cos \tau \sin u_{\oplus} \,.\, \sin d\mathrm{I} \\ & - r \sin \tau \cos u_{\oplus} \,.\, 2e \sin \mathrm{Q} \\ & + r \sin \tau \sin u_{\oplus} \,.\, 2e \cos \mathrm{Q} \\ ds = {} & r \cos \sigma \cos \tau \,.\, \sin du_{\oplus} \\ & + r \cos \sigma \sin p \cos \delta \,.\, \sin d\mathrm{N} \\ & + r \cos \sigma \sin \tau \sin u_{\oplus} \,.\, \sin d\mathrm{I} \\ & - (r \cos \sigma \cos \tau \cos u_{\oplus} + \tfrac{1}{2} s \sin u_{\oplus}) \,.\, 2e \sin \mathrm{Q} \\ & + (r \cos \sigma \cos \tau \sin u_{\oplus} - \tfrac{1}{2} s \cos u_{\oplus}) \,.\, 2e \cos \mathrm{Q} \\ & + s \frac{da}{a}, \end{aligned}$$

where

$$\sin \tau = \frac{r}{s} \sin \mathrm{B},$$

$$\cos \tau = \frac{r}{s} \cos \mathrm{B} \sin (u_{\oplus} - \mathrm{U}),$$

$$\cos \sigma = \cos \mathrm{B} \cos (u_{\oplus} - \mathrm{U}).$$

In the above, e is the assumed unknown eccentricity of the satellite's orbit, and Q is the longitude of the periastron measured from the same node as $u_{\oplus}$.

Sixty-three sets of corrections to the provisional elements have been deduced from 1444 observations made at seven different observatories by fourteen different observers.

In the following tables with these results have been included those obtained by Struve and at Greenwich.

TABLE I.

Longitude of Node.

Observer.	Mean Epoch.	N.		ΔN.	
		Obs.	Comp.	O–C.	*v.*
Bond	1848·3	178°·37±0°·86	179°·87	[−1°·50]	[−1°·16]
O. Struve	1848·6	182·39±1 00	179·90	[+2·49]	[+2·83]
Lassell (Starfield)	1849·8	176·70±0·50	180·01	[−3·31]	[−2·98]
Lassell (Malta)	1852·9	179·02±0·59	180·33	[−1·31]	[−1·00]
O. Struve	1863·6	181·33±0·77	181·54	−0·21	0·00
Lassell and Marth	1864·5	181·65±0·42	181·65	0·00	+0·21
Newcomb	1874·5	183·03±0·14	182·96	+0·07	+0·18
Hall	1876·3	183·47±0·33	183·21	+0·26	+0·34
Holden	1876·5	182·79±0·30	183·24	−0·45	−0·37
Hall	1882·1	184·05±0·15	184·06	−0·01	+0·01

TABLE I.—*continued.*

Observer.	Mean Epoch.	N.		ΔN.	
		Obs.	Comp.	O–C.	*v.*
Hall	1883·8	184°·67±0°·21	184°·31	+0°·36	+0°·36
H. Struve	1887·6	184·48±0·15	184·90	−0·42	−0·47
H. Struve	1889·0	185·05±0·11	185·12	−0·07	−0·14
Parrish	1889·9	184·38±1·21	185·27	−0·89	−0·97
H. Struve	1890·6	185·51±0·12	185·38	+0·13	+0·04
A. Hall, Jr.	1892·0	185·86±0·39	185·60	+0·26	+0·16
H. Struve	1892·6	185·56±0·18	185·70	−0·14	−0·25
Barnard	1892·9	186·51±0·39	185·75	+0·76	+0·65
Barnard	1894·0	185·91±0·36	185·93	−0·02	−0·15
Schaeberle	1895·1	186·09±0·26	186·10	−0·01	−0·15
Barnard	1895·1	186·55±0·23	186·10	+0·45	+0·31
Schaeberle	1895·9	186·49±0·20	186·24	+0·25	+0·10
Schaeberle	1896·8	187·07±0·30	186·40	+0·67	+0·50
Drew	1897·1	186·38±0·33	186·44	−0·06	−0·23
Schaeberle	1897·9	186·81±0·18	186·57	+0·24	+0·06
Brown	1898·0	186·73±0·18	186·59	+0·14	−0·04
Barnard	1898·0	187·22±0·17	186·60	+0·62	+0·44
Drew	1898·8	187·44±0·44	186·72	+0·72	+0·53
Aitken	1898·9	188·33±0·34	186·74	+1·59	+1·39
Barnard	1899·0	186·53±0·14	186·76	−0·23	−0·42
Hussey	1899·0	186·69±0·43	186·76	−0·07	−0·26
Pulkowa Photo.	1899·2	187·00±0·48	186·80	+0·20	0·00
Barnard	1899·9	186·98±0·11	186·92	+0·06	−0·15
See	1900·0	187·26±0·22	186·93	+0·33	+0·12
Hussey	1900·0	186·69±0·40	186·94	−0·25	−0·46
Pulkowa Photo.	1900·1	185·99±0·42	186·96	−0·97	−1·18
Barnard	1900·9	187·05±0·17	187·08	−0·03	−0·25
Pulkowa Photo.	1901·1	188·94±1·35	187·13	+1·81	+1·59
Aitken	1901·8	187·12±0·34	187·25	−0·13	−0·36
Barnard	1902·0	187·69±0·15	187·28	+0·41	+0·17
Lick Photo.	1902·0	188·08±0·29	187·28	+0·80	+0·56
Greenwich Photo.	1902·2	187·38±0·14	187·31	+0·07	−0·17
Barnard	1902·9	188·05±0·20	187·44	+0·61	+0·36
Dinwiddie	1903·0	187·10±0·35	187·46	−0·36	−0·61
Greenwich Photo.	1903·1	187·48±0·21	187·47	+0·01	−0·24
Wirtz	1903·2	186·40±0·91	187·48	−1·08	−1·33
Barnard	1904·0	187·56±0·28	187·62	−0·06	−0·32
Greenwich Photo.	1904·1	187·96±0·15	187·65	+0·31	+0·04

TABLE I.—*continued.*

Observer.	Mean Epoch.	N.		ΔN.	
		Obs.	Comp.	O–C.	*v.*
		°	°	°	°
Barnard	1904·9	187·55±0·30	187·78	−0·23	−0·51
Hammond	1905·0	187·93±0·56	187·81	+0·12	−0·16
Greenwich Photo.	1905·1	188·48±0·19	187·82	+0·66	+0·38
Rice	1905·1	187·40±0·58	187·83	−0·43	−0·71
Barnard	1906·1	189·39±0·47	187·99	+1·40	+1·11
Hammond	1906·1	188·76±0·19	188·00	+0·76	+0·47
Greenwich Photo.	1906·2	188·89±0·19	188·02	+0·87	+0·58
Barnard	1907·1	188·74±0·21	188·18	+0·56	+0·25
Greenwich Photo.	1907·2	188·62±0·20	188·19	+0·43	+0·12
Barnard	1908·1	188·55±0·22	188·36	+0·19	−0·13
Greenwich Photo.	1908·1	188·69±0·28	188·36	+0·33	+0·01
Hammond	1908·1	188·72±0·31	188·36	+0·36	+0·04
Barnard	1909·0	189·40±0·26	188·53	+0·87	+0·53
A. Hall, Jr.	1909·1	189·63±0·36	188·54	+1·09	+0·75
Barnard	1910·1	188·48±0·28	188·71	−0·23	−0·58
A. Hall, Jr.	1910·1	189·16±0·31	188·72	+0·44	+0·09
Greenwich Photo.	1910·2	189·01±0·12	188·74	+0·27	−0·08
Barnard	1911·0	188·11±0·33	188·88	−0·77	−1·13
A. Hall, Jr.	1911·2	188·79±0·30	188·92	−0·13	−0·50
Burton	1911·2	188·30±0·30	188·92	−0·62	−0·99
Burton	1912·1	189·60±0·11	189·09	+0·51	+0·13
Barnard	1912·2	189·60±0·27	189·10	+0·50	+0·12
Barnard	1913·0	189·48±0·23	189·26	+0·22	−0·17
A. Hall, Jr.	1913·0	188·76±0·29	189·26	−0·50	−0·89
Burton	1913·2	190·02±0·17	189·30	+0·72	+0·32
Barnard	1914·0	189·25±0·30	189·44	−0·19	−0·60
Barnard	1915·3	189·47±0·68	189·68	−0·21	−0·64
Barnard	1916·1	190·41±0·27	189·84	+0·57	+0·13
Barnard	1917·0	190·73±0·21	190·00	+0·73	+0·28
Barnard	1918·1	190·64±0·33	190·21	+0·43	−0·04
A. Hall, Jr.	1919·1	190·94±0·29	190·39	+0·55	+0·07
Barnard	1919·1	190·92±0·22	190·40	+0·52	+0·04
A. Hall, Jr.	1920·1	191·94±0·33	190·58	+1·36	+0·86
Barnard	1920·1	190·70±0·27	190·59	+0·11	−0·39
Barnard	1921·1	190·77±0·21	190·78	−0·01	−0·52
A. Hall, Jr.	1921·1	192·20±0·31	190·78	+1·42	+0·90
Barnard	1922·1	191·35±0·27	190·98	+0·37	−0·16
A. Hall, Jr.	1923·3	192·13±0·59	191·20	+0·93	+0·38

TABLE II.

Inclination of Orbit.

Observer.	Mean Epoch.	I. Obs.	I. Comp.	ΔI. O–C.	ΔI. *v.*
Bond	1848·3	125°·05±0°·45	126°·02	[−0°·97]	[−1°·59]
O. Struve	1848·6	126·24±0·48	125·97	[+0·27]	[−0·35]
Lassell (Starfield)	1849·8	126·55±0·21	125·75	[+0·80]	[+0·20]
Lassell (Malta)	1852·9	126·21±0·39	125·20	[+1·01]	[+0·46]
O. Struve	1863·6	124·22±0·32	123·35	+0·87	+0·54
Lassell and Marth	1864·5	124·19±0·28	123·20	+0·99	+0·63
Newcomb	1874·5	121·70±0·10	121·57	+0·13	−0·08
Hall	1876·3	121·64±0·19	121·29	+0·35	+0·17
Holden	1876·5	121·04±0·18	121·26	−0·22	−0·40
Hall	1882·1	120·03±0·11	120·40	−0·37	−0·46
Hall	1883·8	120·13±0·23	120·14	−0·01	−0·08
H. Struve	1887·6	119·38±0·13	119·58	−0·20	−0·22
H. Struve	1889·0	119·53±0·09	119·38	+0·15	+0·15
Parrish	1889·9	118·71±1·17	119·25	−0·54	−0·53
H. Struve	1890·6	119·26±0·10	119·16	+0·10	+0·12
A. Hall, Jr.	1892·0	118·93±0·36	118·96	−0·03	+0·01
H. Struve	1892·6	119·06±0·15	118·88	+0·18	+0·23
Barnard	1892·9	119·52±0·41	118·83	+0·69	+0·74
Barnard	1894·0	118·80±0·37	118·68	+0·12	+0·19
Schaeberle	1895·1	118·71±0·21	118·54	+0·17	+0·25
Barnard	1895·1	118·65±0·22	118·54	+0·11	+0·19
Schaeberle	1895·9	118·11±0·24	118·43	−0·32	−0·23
Schaeberle	1896·8	118·59±0·29	118·30	+0·29	+0·40
Drew	1897·1	118·29±0·29	118·27	+0·02	+0·13
Schaeberle	1897·9	118·04±0·15	118·16	−0·12	0·00
Brown	1898·0	117·79±0·15	118·15	−0·36	−0·24
Barnard	1898·0	118·19±0·14	118·14	+0·05	+0·17
Drew	1898·8	117·79±0·38	118·05	−0·26	−0·13
Aitken	1898·9	116·94±0·27	118·03	−1·09	−0·96
Barnard	1899·0	118·02±0·12	118·02	0·00	+0·13
Hussey	1899·0	117·70±0·47	118·02	−0·32	−0·19
Pulkowa Photo.	1899·2	118·74±0·41	117·99	+0·75	+0·88
Barnard	1899·9	117·94±0·11	117·90	+0·04	+0·18
See	1900·0	117·87±0·20	117·89	−0·02	+0·13
Hussey	1900·0	117·97±0·39	117·88	+0·09	+0·24

TABLE II.—*continued.*

Observer.	Mean Epoch.	I. Obs.	I. Comp.	ΔI. O–C.	ΔI. *v.*
		°	°	°	°
Pulkowa Photo.	1900·1	118·93±0·45	117·87	+1·06	+1·21
Barnard	1900·9	117·77±0·14	117·77	0·00	+0·16
Pulkowa Photo.	1901·1	118·22±1·14	117·74	+0·48	+0·64
Aitken	1901·8	116·85±0·30	117·65	−0·80	−0·63
Barnard	1902·0	117·87±0·14	117·63	+0·24	+0·41
Lick Photo.	1902·0	117·06±0·27	117·63	−0·57	−0·40
Greenwich Photo.	1902·2	117·45±0·13	117·61	−0·16	+0·01
Barnard	1902·9	117·21±0·17	117·51	−0·30	−0·12
Dinwiddie	1903·0	117·79±0·35	117·50	+0·29	+0·47
Greenwich Photo.	1903·1	117·48±0·20	117·49	−0·01	+0·17
Wirtz	1903·2	117·54±0·84	117·48	+0·06	+0·24
Barnard	1904·0	117·37±0·24	117·38	−0·01	+0·18
Greenwich Photo.	1904·1	117·22±0·13	117·36	−0·14	+0·06
Barnard	1904·9	116·80±0·27	117·27	−0·47	−0·26
Hammond	1905·0	116·42±0·51	117·25	−0·83	−0·62
Greenwich Photo.	1905·1	117·15±0·17	117·24	−0·09	+0·12
Rice	1905·1	117·02±0·42	117·24	−0·22	−0·01
Barnard	1906·1	117·00±0·43	117·12	−0·12	+0·10
Hammond	1906·1	116·40±0·19	117·12	−0·72	−0·50
Greenwich Photo.	1906·2	116·56±0·15	117·11	−0·55	−0·33
Barnard	1907·1	116·72±0·19	117·00	−0·28	−0·05
Greenwich Photo.	1907·2	116·72±0·18	116·99	−0·27	−0·04
Barnard	1908·1	116·63±0·19	116·88	−0·25	−0·01
Greenwich Photo.	1908·1	116·22±0·24	116·88	−0·66	−0·42
Hammond	1908·1	117·27±0·34	116·87	+0·40	+0·64
Barnard	1909·0	116·22±0·22	116·77	−0·55	−0·30
A. Hall, Jr.	1909·1	116·47±0·35	116·76	−0·29	−0·04
Barnard	1910·1	116·49±0·27	116·65	−0·16	+0·10
A. Hall, Jr.	1910·1	115·74±0·30	116·64	−0·90	−0·63
Greenwich Photo.	1910·2	116·31±0·11	116·63	−0·32	−0·05
Barnard	1911·0	115·37±0·27	116·54	−1·17	−0·90
A. Hall, Jr.	1911·2	115·70±0·28	116·52	−0·82	−0·54
Burton	1911·2	116·62±0·30	116·52	+0·10	+0·38
Burton	1912·1	116·15±0·10	116·41	−0·26	+0·03
Barnard	1912·2	116·22±0·24	116·40	−0·18	+0·11

TABLE II.—*continued.*

Observer.	Mean Epoch.	I.		ΔI.	
		Obs.	Comp.	O–C.	*v.*
		°	°	°	°
Barnard	1913·0	115·73±0·21	116·31	−0·58	−0·28
A. Hall, Jr.	1913·0	116·21±0·32	116·31	−0·10	+0·20
Burton	1913·2	115·78±0·16	116·28	−0·50	−0·20
Barnard	1914·0	115·84±0·29	116·20	−0·36	−0·05
Barnard	1915·3	115·93±0·49	116·06	−0·13	+0·19
Barnard	1916·1	115·49±0·26	115·97	−0·48	−0·15
Barnard	1917·0	115·98±0·19	115·87	+0·11	+0·45
Barnard	1918·1	115·78±0·29	115·75	+0·03	+0·38
A. Hall, Jr.	1919·1	115·12±0·28	115·65	−0·53	−0·17
Barnard	1919·1	115·72±0·20	115·65	+0·07	+0·43
A. Hall, Jr.	1920·1	115·49±0·31	115·55	−0·06	+0·31
Barnard	1920·1	115·14±0·23	115·55	−0·41	−0·04
Barnard	1921·1	115·06±0·20	115·45	−0·39	−0·01
A. Hall, Jr.	1921·1	115·06±0·29	115·44	−0·38	0·00
Barnard	1922·1	114·52±0·21	115·34	−0·82	−0·43
A. Hall, Jr.	1923·3	115·21±0·48	115·23	−0·02	+0·38

The Pole of Neptune's Equator.

The computed values of N and I, column four of Tables I. and II., are obtained from the formulæ below on the following assumptions: That the plane of Neptune's equator intersects the plane of the Earth's equator of 1900·0 in longitude 23°·56 and makes an angle with it of 49°·24; that the plane of the orbit of Neptune's satellite at 1900·0 intersects the plane of Neptune's equator in longitude 151°·72 measured on the Earth's equator of 1900·0 from the equinox of that date to the node of Neptune's equator and thence along Neptune's equator, and that it makes an angle of 161°·24 with the plane of Neptune's equator; and that while the inclination, 161°·24, is constant, the longitude of the node, 151°·72, increases 60°·08 per century.

Putting

N=Longitude of ascending node of orbit of Neptune's satellite on the Earth's equator,

I=Inclination of the orbit of Neptune's satellite to the Earth's equator,

N_0=Longitude of node of Neptune's equator on the Earth's equator,

I_0=Inclination of Neptune's equator to the Earth's equator,

θ=Longitude of ascending node of orbit of Neptune's satellite on Neptune's equator measured from the node of Neptune's equator on the Earth's equator,

$\delta\theta$=Increase of θ in a century,

γ=Inclination of the orbit of Neptune's satellite to Neptune's equator,

ψ=Distance from ascending node of orbit of Neptune's satellite on the Earth's equator to the ascending node on Neptune's equator,

we have

$$\begin{aligned}
\sin I \sin (N-N_0) &= \sin \gamma \sin \theta, \\
\sin I \cos (N-N_0) &= \cos \gamma \sin I_0 + \sin \gamma \cos I_0 \cos \theta, \\
\cos I &= \cos \gamma \cos I_0 - \sin \gamma \sin I_0 \cos \theta, \\
\sin I \sin \psi &= \sin I_0 \sin \theta, \\
\sin I \cos \psi &= \sin \gamma \cos I_0 + \cos \gamma \sin I_0 \cos \theta,
\end{aligned}$$

and to reduce from the equinox of 1900·0 to that of date:

$$\begin{aligned}
\Delta N_0 &= (46''\cdot08 - 20''\cdot05 \cos N_0 \cot I_0)(T-1900\cdot0), \\
\Delta I_0 &= -20''\cdot05(T-1900\cdot0) \sin N_0, \\
\Delta\theta &= 20''\cdot05(T-1900\cdot0) \cos N_0 \csc I_0.
\end{aligned}$$

Column five of Table I. and column five of Table II. respectively give the differences between the N and I thus computed and those derived from the observations of the satellite. Each pair of differences in N and I for a given date furnishes two equations of condition between the corrections to the assumed quantities, N_0, I_0, θ, $\delta\theta$, and γ. These equations have the form:

$$\begin{aligned}
dN_0 - \cot I \sin (N-N_0) dI_0 + \csc I \sin \gamma \cos \psi \,(d\theta_0 + t \,.\, d\delta\theta) \\
+ \csc I \sin \psi d\gamma = \Delta N(O-C), \\
\cos (N-N_0) dI_0 - \sin \gamma \sin \psi (d\theta_0 + t \,.\, d\delta\theta) + \cos \psi d\gamma = \Delta I(O-C),
\end{aligned}$$

where t is the fraction of a century from 1900·0.

Because of the large value of several of the early residuals it was decided not to use those before 1860.

In forming the normal equations, weights were assigned to the equations of condition in accordance with the probable errors of N and I respectively, weight unity corresponding to a probable error of 1°.

The results of the solution of the normal equations are:

$$\begin{aligned}
dN_0 &= +1^\circ\cdot593 \pm 2^\circ\cdot42, & N_0 &= 25^\circ\cdot153 \pm 2^\circ\cdot42, \\
dI_0 &= -0\cdot588 \pm 1\cdot38, & I_0 &= 48\cdot652 \pm 1\cdot38, \\
d\theta_0 &= -1\cdot145 \pm 1\cdot86, & \theta_0 &= 127\cdot015 \pm 1\cdot86, \\
d\delta\theta &= +1\cdot414 \pm 6\cdot98, & \delta\theta &= 61\cdot494 \pm 6\cdot98, \\
d\gamma &= -1\cdot295 \pm 2\cdot27, & \gamma &= 159\cdot945 \pm 2\cdot27,
\end{aligned}$$

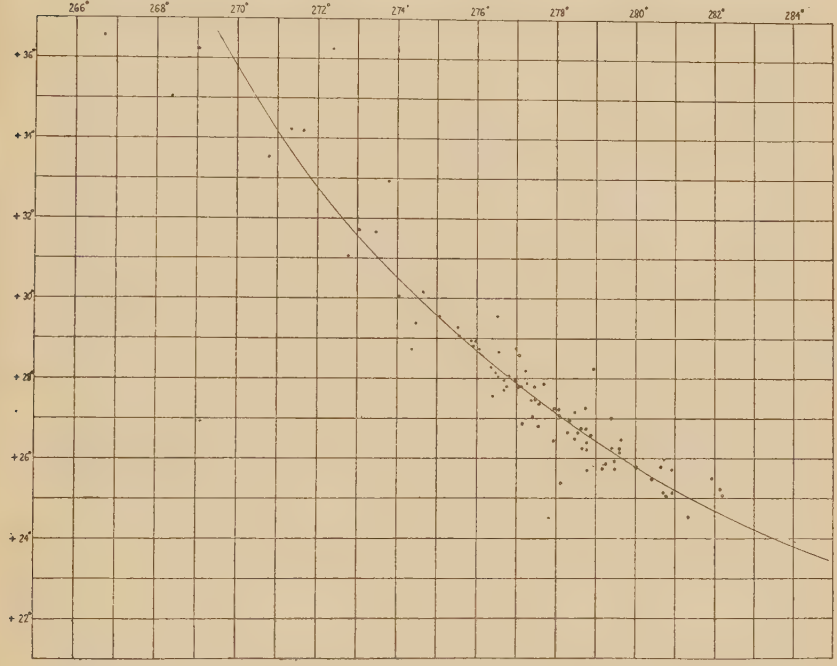
giving for the position of the north pole of Neptune's equator

$$\left.\begin{aligned} A &= 295^\circ\cdot2 \\ D &= +41\cdot3 \end{aligned}\right\} 1900\cdot0,$$

the period of the revolution of the pole of the satellite orbit around the pole of Neptune's equator, 585 years, and the radius of the circle described by the pole of the satellite's orbit, 20°·1.*

* Previous determinations of this quantity are: by S. J. Brown in 1900, 18°; by Dyson and Edney in 1905, 22°; and by David Gibb in 1909, 21°·2.

Table III. gives the values of N and I at ten-year intervals during the period of revolution, and the diagram below gives the observational positions of the pole of the satellite's orbit together with definitive path from 1848 to 1938.



Observational Positions of Pole of Satellite's Orbit with Definitive Path from 1848 to 1938.

Longitude and Motion of Satellite.

In column two of Table IV. are collected the values of the corrections to the assumed mean angular distance of the satellite in its orbit from the ascending node of the orbit on the Earth's equator,

$$u_{\oplus}=234^{\circ}\cdot42+61^{\circ}\cdot25748(t_{d}-1890 \text{ Jan. } 0, \text{ G.M.N.}).$$

From the data of the preceding section can be obtained the distance, ψ, from the ascending node of the orbit of the satellite on the Earth's equator to the ascending node on Neptune's equator. If this is expressed in the form

$$\psi=47^{\circ}\cdot31-0^{\circ}\cdot00140(t_{d}-1890 \text{ Jan. } 0, \text{ G.M.N.})+d\psi,$$

the values of $d\psi$ are those given in column three of Table IV.

TABLE III.

Values of N, I, *and* ψ *at Ten-Year Intervals.*

[Mean Equator and Equinox of 1900·0.]

Date.	N.		I.		ψ.		Date.	N.		I.		ψ.	
	°		°		°			°		°		°	
1840	179·2		128·3		73·1		2140	231·6		129·9		282·8	
		+0·9		−2·0		−5·2			+0·5		+2·1		−5·5
1850	180·1		126·3		67·9		2150	232·1		132·0		277·3	
		+1·1		−1·9		−5·2			+0·2		+2·0		−5·6
1860	181·2		124·4		62·7		2160	232·3		134·0		271·7	
		+1·3		−1·8		−5·1			−0·1		+2·2		−5·8
1870	182·5		122·6		57·6		2170	232·2		136·2		265·9	
		+1·4		−1·7		−5·0			−0·3		+2·1		−6·1
1880	183·9		120·9		52·6		2180	231·9		138·3		259·8	
		+1·5		−1·7		−5·0			−0·7		+2·0		−6·3
1890	185·4		119·2		47·6		2190	231·2		140·3		253·5	
		+1·7		−1·4		−5·0			−1·2		+2·0		−6·7
1900	187·1		117·8		42·6		2200	230·0		142·3		246·8	
		+1·8		−1·4		−4·9			−1·6		+1·9		−7·0
1910	188·9		116·4		37·7		2210	228·4		144·2		239·8	
		+1·9		−1·2		5·0			−2·1		+1·7		−7·5
1920	190·8		115·2		32·7		2220	226·3		145·9		232·3	
		+2·0		−1·1		−4·9			−2·5		+1·6		−7·9
1930	192·8		114·1		27·8		2230	223·8		147·5		224·4	
		+2·1		−0·9		−5·0			−3·1		+1·4		−8·4
1940	194·9		113·2		22·8		2240	220·7		148·9		216·0	
		+2·2		−0·7		−4·9			−3·5		+1·1		−8·8
1950	197·1		112·5		17·9		2250	217·2		150·0		207·2	
		+2·2		−0·6		−4·9			−3·9		+0·8		−9·2
1960	199·3		111·9		13·0		2260	213·3		150·8		198·0	
		+2·2		−0·4		−5·0			−4·3		+0·5		−9·5
1970	201·5		111·5		8·0		2270	209·0		151·3		188·5	
		+2·3		−0·2		−5·0			−4·4		−0·1		−9·6
1980	203·8		111·3		3·0		2280	204·6		151·4		178·9	
		+2·2		0·0		−4·9			−4·3		−0·2		−9·6
1990	206·0		111·3		358·1		2290	200·3		151·2		169·3	
		+2·3		+0·2		−4·9			−4·2		−0·6		−9·4
2000	208·3		111·5		353·2		2300	196·1		150·6		159·9	
		+2·2		+0·3		−5·0			−3·8		−0·9		−9·1
2010	210·5		111·8		348·2		2310	192·3		149·7		150·8	
		+2·2		+0·5		−5·0			−3·5		−1·1		−8·8
2020	212·7		112·3		343·2		2320	188·8		148·6		142·0	
		+2·2		+0·7		−4·9			−2·9		−1·4		−8·2
2030	214·9		113·0		338·3		2330	185·9		147·2		133·8	
		+2·1		+0·9		−4·9			−2·4		−1·6		−7·8
2040	217·0		113·9		333·4		2340	183·5		145·6		126·0	
		+2·0		+1·0		−5·0			−2·0		−1·8		−7·4
2050	219·0		114·9		328·4		2350	181·5		143·8		118·6	
		+1·9		+1·2		−4·9			−1·5		−2·0		−7·0
2060	220·9		116·1		323·5		2360	180·0		141·8		111·6	
		+1·9		+1·3		−5·0			−1·0		−2·0		−6·5
2070	222·8		117·4		318·5		2370	179·0		139·8		105·1	
		+1·7		+1·5		−4·9			−0·7		−2·0		−6·3
2080	224·5		118·9		313·6		2380	178·3		137·8		98·8	
		+1·6		+1·6		−5·0			−0·3		−2·1		−6·0
2090	226·1		120·5		308·6		2390	178·0		135·7		92·8	
		+1·4		+1·7		−5·0			0·0		−2·1		−5·8
2100	227·5		122·2		303·6		2400	178·0		133·6		87·0	
		+1·3		+1·8		−5·1			+0·3		−2·1		−5·6
2110	228·8		124·0		298·5		2410	178·3		131·5		81·4	
		+1·2		+1·9		−5·2			+0·5		−2·1		−5·4
2120	230·0		125·9		293·3		2420	178·8		129·4		76·0	
		+0·9		+2·0		−5·2			+0·8		−2·0		−5·3
2130	230·9		127·9		288·1		2430	179·6		127·4		70·7	
		+0·7		+2·0		−5·3			+1·0		−1·9		−5·1
2140	231·6		129·9		282·8		2440	180·6		125·5		65·6	

Subtracting ψ from $u_\oplus$, there results the mean angular distance of the satellite in its orbit from the ascending node of the orbit on Neptune's equator,

$$u_{\overset{\circ}{\Psi}}=187^\circ\cdot11+61^\circ\cdot25888(t_d-1890\text{ Jan. }0,\text{ G.M.N.})+du_{\overset{\circ}{\Psi}}.$$

The values of $du_{\overset{\circ}{\Psi}}=du_\oplus-d\psi$ are given in column four of Table IV.

By the method of least squares the following value is obtained for $du_{\overset{\circ}{\Psi}}$:

$$du_{\overset{\circ}{\Psi}}=-0^\circ\cdot190\pm0^\circ\cdot019+(0^\circ\cdot0000694\pm0^\circ\cdot0000052)(t_d-1890\text{ Jan. }0,\text{ G.M.N.}),$$

and the residuals, v, are given in column five of Table IV. The definitive value of $u_{\overset{\circ}{\Psi}}$ is therefore

$$u_{\overset{\circ}{\Psi}}=186^\circ\cdot92+61^\circ\cdot2589494(t_d-1890\text{ Jan. }0,\text{ G.M.N.}).$$

To obtain the daily sidereal motion of the satellite, it is necessary to subtract from the daily motion of $u_{\overset{\circ}{\Psi}}$ just given, the daily motion of θ multiplied by cos $(180^\circ-\gamma)$ or $0^\circ\cdot0015815$. We thus have:

Daily sidereal motion$=61^\circ\cdot2573679$.

This value differs only $0^\circ\cdot00008$ from the first determination of this motion, that by S. J. Brown twenty-five years ago, when the position of Neptune's equator could be given only with great uncertainty.

TABLE IV.

Longitude in Orbit and Mean Distance.

Mean Epoch.	$du_\oplus$.	$d\psi$.	$du_{\overset{\circ}{\Psi}}$.	v.	a.
	° °	°	°	°	″ ″
1848·3	+1·98±0·98	+0·47	[+1·51]	[+2·76]	
1852·9	−0·13±0·58	+0·40	[−0·53]	[+0·60]	
1864·5	+0·59±0·51	+0·31	+0·28	+1·12	
1874·5	+0·03±0·13	+0·29	−0·26	+0·32	16·275±0·018
1876·3	−0·41±0·27	+0·29	−0·70	−0·16	16·482±0·033
1876·5	0·00±0·24	+0·29	−0·29	+0·24	16·598±0·035
1882·1	+0·07±0·12	+0·31	−0·24	+0·15	16·368±0·022
1883·8	+0·53±0·19	+0·32	+0·21	+0·56	16·263±0·028
1887·6	−0·45±0·11	+0·34	−0·79	−0·54	16·285±0·028
1889·0	0·00±0·08	+0·35	−0·35	−0·13	16·272±0·019
1889·9	−0·20±0·68	+0·36	−0·56	−0·37	16·166±0·274
1890·6	+0·05±0·08	+0·36	−0·31	−0·14	16·253±0·021
1892·0	+0·40±0·25	+0·37	+0·03	+0·17	16·579±0·068
1892·6	+0·17±0·12	+0·38	−0·21	−0·09	16·293±0·032
1892·9	+0·70±0·25	+0·38	+0·32	+0·44	16·430±0·064

TABLE IV.—*continued.*

Mean Epoch.	$du_{\oplus}$.	$d\psi$.	$du_{\frac{11}{9}}$.	r.	a.
	° ′	°	°	°	″
1894·0	+0·51±0·26	+0·39	+0·12	+0·21	16·282±0·051
1895·1	+0·34±0·15	+0·39	−0·05	+0·01	16·287±0·046
1895·1	+0·12±0·14	+0·39	−0·27	−0·21	16·313±0·041
1895·9	+0·43±0·14	+0·40	+0·03	+0·07	16·264±0·042
1896·8	+0·44±0·17	+0·41	+0·03	+0·05	16·354±0·052
1897·1	−0·02±0·18	+0·41	−0·43	−0·42	16·223±0·057
1897·9	+0·53±0·10	+0·42	+0·11	+0·10	16·393±0·029
1898·0	+0·46±0·10	+0·42	+0·04	+0·03	16·272±0·029
1898·0	+0·35±0·09	+0·42	−0·07	−0·08	16·236±0·028
1898·8	+0·57±0·22	+0·43	+0·14	+0·11	16·509±0·073
1898·9	+0·07±0·19	+0·43	−0·36	−0·40	16·039±0·059
1899·0	+0·22±0·07	+0·43	−0·21	−0·25	16·220±0·023
1899·0	+0·24±0·25	+0·43	−0·19	−0·23	16·380±0·062
1899·2	+1·10±0·26	+0·43	+0·67	+0·63	16·170±0·078
1899·9	+0·54±0·06	+0·44	+0·10	+0·04	16·284±0·018
1900·0	+0·78±0·12	+0·44	+0·34	+0·28	16·544±0·036
1900·0	−0·09±0·20	+0·44	−0·53	−0·59	16·696±0·062
1900·1	−0·16±0·22	+0·44	−0·60	−0·67	16·157±0·070
1900·9	+0·23±0·08	+0·44	−0·21	−0·29	16·311±0·028
1901·1	+1·18±0·86	+0·45	+0·73	+0·65	15·925±0·267
1901·8	+0·76±0·18	+0·45	+0·31	+0·20	16·217±0·051
1902·0	+0·48±0·08	+0·46	+0·02	−0·09	16·365±0·024
1902·0	+0·43±0·15	+0·46	−0·03	−0·14	16·186±0·044
1902·2	+0·95±0·07	+0·46	+0·49	+0·37	16·182±0·024
1902·9	+0·56±0·10	+0·46	+0·10	−0·04	16·365±0·034
1903·0	+0·47±0·18	+0·46	+0·01	−0·13	16·328±0·051
1903·1	+0·66±0·11	+0·46	+0·20	+0·06	16·179±0·032
1903·2	−1·14±0·46	+0·47	−1·61	−1·75	16·248±0·143
1904·0	+0·45±0·14	+0·47	−0·02	−0·18	16·341±0·047
1904·1	+0·84±0·08	+0·47	+0·37	+0·20	16·238±0·024
1904·9	+0·61±0·15	+0·48	+0·13	−0·05	16·427±0·047
1905·0	+0·60±0·28	+0·48	+0·12	−0·07	16·235±0·086
1905·1	+0·80±0·09	+0·48	+0·32	+0·13	16·178±0·030
1905·1	+0·52±0·28	+0·48	+0·04	−0·15	16·667±0·083
1906·1	+0·83±0·24	+0·49	+0·34	+0·12	16·554±0·090

TABLE IV.—*continued.*

Mean Epoch.	$du_{\oplus}$.	$d\psi$.	$du_{\frac{1}{2}\pi}$.	v.	a.
1906·1	+0°·94±0°·10	+0°·49	+0°·45	+0°·23	16″·209±0″·029
1906·2	+0·97±0·09	+0·49	+0·48	+0·26	16·185±0·031
1907·1	+0·72±0·10	+0·50	+0·22	−0·02	16·267±0·034
1907·2	+1·02±0·10	+0·50	+0·52	+0·28	16·279±0·031
1908·1	+0·48±0·11	+0·51	−0·03	−0·30	16·360±0·036
1908·1	+0·74±0·14	+0·51	+0·23	−0·04	16·020±0·046
1908·1	+0·94±0·17	+0·51	+0·43	+0·16	16·445±0·044
1909·0	+0·76±0·13	+0·53	+0·23	−0·06	16·316±0·042
1909·1	+1·23±0·18	+0·53	+0·70	+0·41	16·585±0·055
1910·1	+0·93±0·14	+0·54	+0·39	+0·07	16·323±0·044
1910·1	+0·51±0·16	+0·54	−0·03	−0·35	16·346±0·046
1910·2	+1·06±0·06	+0·54	+0·52	+0·20	16·217±0·018
1911·0	+0·88±0·16	+0·54	+0·34	0·00	16·411±0·054
1911·2	+1·09±0·16	+0·55	+0·54	+0·20	16·380±0·037
1911·2	+0·98±0·15	+0·55	+0·43	+0·08	16·355±0·044
1912·1	+0·96±0·06	+0·56	+0·40	+0·03	16·383±0·016
1912·2	+0·97±0·13	+0·56	+0·41	+0·04	16·375±0·044
1913·0	+0·84±0·12	+0·57	+0·27	−0·12	16·298±0·037
1913·0	+0·96±0·16	+0·57	+0·39	0·00	16·468±0·046
1913·2	+0·72±0·08	+0·57	+0·15	−0·25	16·315±0·026
1914·0	+1·05±0·16	+0·58	+0·47	+0·05	16·416±0·051
1915·3	+0·60±0·39	+0·59	+0·01	−0·44	16·512±0·104
1916·1	+1·20±0·14	+0·60	+0·60	+0·13	16·346±0·049
1917·0	+1·25±0·12	+0·61	+0·64	+0·15	16·293±0·036
1918·1	+1·20±0·19	+0·62	+0·58	+0·06	16·372±0·059
1919·1	+0·88±0·17	+0·63	+0·25	−0·30	16·406±0·047
1919·1	+1·09±0·13	+0·63	+0·46	−0·09	16·395±0·037
1920·1	+1·22±0·21	+0·64	+0·58	+0·01	16·422±0·052
1920·1	+0·64±0·16	+0·64	0·00	−0·57	16·411±0·046
1921·1	+1·04±0·12	+0·66	+0·38	−0·22	16·333±0·039
1921·1	+1·80±0·23	+0·66	+1·14	+0·54	16·440±0·046
1922·1	+1·18±0·17	+0·67	+0·51	−0·11	16·399±0·044
1923·3	+1·04±0·43	+0·68	+0·36	−0·29	16·447±0·067

Mean Distance of Satellite and Mass of Neptune.

In column six of Table IV. are collected eighty determinations of the mean distance of the satellite from its primary made from 1874 to 1923. These determinations range in value from 15″·9 to 16″·7, and indicate that, in addition to the accidental errors of the determinations, there are evident systematic differences between the determinations of different observers. Weighting the various determinations according to their probable errors, giving weight unity to a probable error of 0″·1, the following results are obtained by observers :—

Hall . . .	3	determinations,	1876–1884, $a=16''\cdot360\pm0''\cdot015$
Struve . . .	4	,,	1887–1893, $a=16\cdot271\pm0\cdot012$
Schaeberle . .	4	,,	1895–1898, $a=16\cdot336\pm0\cdot019$
Hammond . .	3	,,	1905–1908, $a=16\cdot276\pm0\cdot023$
Burton . . .	3	,,	1911–1913, $a=16\cdot363\pm0\cdot013$
Barnard (Lick) .	3	,,	1892–1895, $a=16\cdot322\pm0\cdot029$
Barnard (Yerkes)	25	,,	1898–1922, $a=16\cdot321\pm0\cdot009$
A. Hall, Jr. .	8	,,	1909–1923, $a=16\cdot425\pm0\cdot016$
Greenwich photo.	8	,,	1902–1910, $a=16\cdot201\pm0\cdot013$
All visual . .	66	,,	1874–1923, $a=16\cdot333\pm0\cdot007$
All photographic	11	,,	1899–1910, $a=16\cdot199\pm0\cdot010$
Final mean . .	77	,,	1874–1923, $a=16\cdot289\pm0\cdot006$

From the last three values of a we obtain for the reciprocal of the mass of Neptune,

From Visual Observations,

$$19176\pm25.$$

From Photographic Observations,

$$19655\pm36.$$

From all Observations combined,

$$19331\pm21.$$

The value used by Newcomb, in his tables of Uranus (*Astronomical Papers of the American Ephemeris*, vol. vii.), is 19314.

The Eccentricity of Satellite's Orbit and Longitude of Periastron.

Columns two and three of Table V. contain the values of the eccentricity, e, and the longitude of periastron, Q, of the satellite orbit. Weighting the values of e according to their probable errors, we obtain as the weighted mean :

$$e_0=0\cdot0049\pm0\cdot0002.$$

This result must not be taken as definitely demonstrating that the orbit of the satellite is elliptical. An eccentricity of this size would

produce a difference between the periastron and apastron distances of less than 0″·2. Even though the orbit is circular, every set of elements deduced as above may be expected to exhibit an eccentricity due to the unavoidable errors of observation. The confirmation of the ellipticity of the orbit must be sought in the accordance of the various values of Q. During the entire period of observation there is not a single opposition when there are three independent determinations of elements that the range of the values of Q is not over 100°.

TABLE V.

Eccentricity and Longitude of Periastron.

Mean Epoch.	e.	Q.	ψ.	ΔQ.	Q_F.	v.
		° °	°	°	°	°
1874·5	0·0088±0·0020	182± 7	55·5	−14·7	141	− 53·1
1876·3	0·0090±0·0031	202±14	54·6	13·7	161	− 28·5
1876·5	0·0051±0·0020	88±36	54·5	13·6	47	−142·0
1882·1	0·0034±0·0014	124±24	51·7	10·3	83	− 91·7
1883·8	0·0100±0·0021	150±11	50·8	− 9·4	109	− 61·3
1887·6	0·0050±0·0013	267±13	48·9	− 7·2	225	+ 64·4
1889·0	0·0075±0·0011	248± 8	48·2	6·4	206	+ 49·0
1889·9	0·0102±0·0128	156·3±41·7	47·7	5·8	114·4	− 40·3
1890·6	0·0072±0·0009	271± 6	47·4	5·4	229	+ 76·0
1892·0	0·0074±0·0033	126·6±16·0	46·7	− 4·6	84·5	− 64·9
1892·6	0·0083±0·0014	248± 8	46·4	− 4·3	206	+ 58·1
1892·9	0·0022±0·0028	177·4±72·0	46·2	4·1	135·3	[− 11·7]
1894·0	0·0065±0·0026	107·0±19·6	45·7	3·5	64·8	− 79·5
1895·1	0·0065±0·0017	149·5±13·5	45·1	2·9	107·3	− 34·3
1895·1	0·0025±0·0016	78·2±35·5	45·1	− 2·8	35·9	−105·7
1895·9	0·0031±0·0025	141·6±19·4	44·7	− 2·4	99·3	− 40·1
1896·8	0·0039±0·0023	91·5±26·2	44·2	1·8	49·1	− 88·0
1897·1	0·0047±0·0022	148·0±22·8	44·1	1·7	105·6	− 30·8
1897·9	0·0047±0·0013	77·7±11·4	43·7	1·2	35·2	− 99·2
1898·0	0·0059±0·0009	0·0±12·2	43·7	− 1·2	317·5	−176·6
1898·0	0·0048±0·0011	131·6±10·6	43·6	− 1·1	89·1	− 44·9
1898·8	0·0050±0·0021	4·6±35·3	43·3	0·7	322·0	−170·1
1898·9	0·0126±0·0024	85·5± 6·4	43·2	0·6	42·9	− 88·9
1899·0	0·0038±0·0008	230·4±13·3	43·1	0·6	187·9	+ 56·3
1899·0	0·0016±0·0024	50·2±98·5	43·1	− 0·6	7·7	[−123·9]
1899·2	0·0024±0·0029	240·2±67·0	43·0	− 0·5	197·7	[+ 66·6]
1899·9	0·0033±0·0007	231·1±12·7	42·7	− 0·1	188·5	+ 59·2
1900·0	0·0038±0·0012	341·6±21·9	42·7	0·0	298·9	−190·2
1900·0	0·0048±0·0019	30·3±32·1	42·6	0·0	347·7	−141·1
1900·1	0·0108±0·0030	302·5±10·7	42·6	+ 0·1	259·8	+131·1

TABLE V.—*continued.*

Mean Epoch.	*e.*	Q.	ψ.	ΔQ.	Q_F.	*v.*
1900·9	0·0076±0·0010	249°·9± 6°·5	42°·2	+ 0°·5	207°·2	+ 80°·4
1901·1	0·0132±0·0094	339·6±39·9	42·1	0·6	296·9	+170·7
1901·8	0·0068±0·0017	147·0±17·5	41·7	1·1	104·2	− 20·1
1902·0	0·0049±0·0007	192·9±12·1	41·6	1·2	150·1	+ 26·2
1902·0	0·0123±0·0019	289·0± 6·9	41·6	+ 1·2	246·2	+122·4
1902·2	0·0006±0·0009	325·1±74·1	41·6	+ 1·2	282·3	[+158·8]
1902·9	0·0057±0·0010	224·3±12·0	41·2	1·7	181·4	+ 59·9
1903·0	0·0147±0·0021	73·8± 7·2	41·1	1·7	31·0	− 90·3
1903·1	0·0009±0·0010	198·0±86·9	41·1	1·8	155·1	[+ 34·1]
1903·2	0·0206±0·0058	257·7±12·8	41·0	+ 1·8	214·9	+ 94·0
1904·0	0·0067±0·0014	213·5±14·5	40·6	+ 2·3	170·6	+ 51·7
1904·1	0·0008±0·0010	97·5±50·2	40·6	2·4	54·5	[− 64·0]
1904·9	0·0059±0·0017	234·7±15·9	40·2	2·8	191·7	+ 75·2
1905·0	0·0050±0·0035	58·8±28·1	40·1	2·9	15·8	−100·4
1905·1	0·0023±0·0010	210·7±29·3	40·1	+ 2·9	167·7	+ 51·8
1905·1	0·0084±0·0037	127·3±19·6	40·1	+ 3·0	84·2	− 31·7
1906·1	0·0096±0·0030	118·7±12·8	39·6	3·5	75·6	− 37·9
1906·1	0·0044±0·0010	332·9±15·3	39·6	3·5	289·8	−183·6
1906·2	0·0022±0·0010	127·4±25·7	39·5	3·6	84·3	− 28·9
1907·1	0·0026±0·0013	110·6±22·5	39·1	+ 4·1	67·4	− 43·5
1907·2	0·0013±0·0013	250·8±42·1	39·0	+ 4·1	207·7	+ 97·0
1908·1	0·0056±0·0009	193·5±14·1	38·6	4·7	150·2	+ 41·8
1908·1	0·0070±0·0018	241·6±10·1	38·6	4·7	198·3	+ 90·0
1908·1	0·0022±0·0017	339·1±51·1	38·6	4·7	295·8	[−172·5]
1909·0	0·0078±0·0015	218·2±10·4	38·1	+ 5·2	174·9	+ 69·0
1909·1	0·0030±0·0018	156·0±43·1	38·1	+ 5·3	112·6	+ 6·8
1910·1	0·0090±0·0015	200·3±10·7	37·6	5·8	156·9	+ 53·6
1910·1	0·0095±0·0019	36·0±10·7	37·6	5·8	352·6	−110·5
1910·2	0·0027±0·0008	84·7±11·5	37·5	5·9	41·3	− 61·7
1911·0	0·0066±0·0020	226·9±14·0	37·1	+ 6·3	183·5	+ 82·5
1911·2	0·0046±0·0017	114·3±16·7	37·0	+ 6·5	70·8	− 29·7
1911·2	0·0092±0·0018	65·7± 9·3	37·0	6·5	22·2	− 78·3
1912·1	0·0056±0·0006	108·8± 6·1	36·6	7·0	65·2	− 32·8
1912·2	0·0044±0·0014	195·9±21·3	36·5	7·0	152·4	+ 54·4
1913·0	0·0066±0·0013	201·2±11·6	36·1	+ 7·5	157·6	+ 61·9

TABLE V.—*continued.*

Mean Epoch.	e.	Q.	ψ.	ΔQ.	Q_F.	v.
1913·0	0·0105±0·0020	53°·1± 8°·3	36°·1	+ 7°·5	9°·5	− 86°·2
1913·2	0·0015±0·0008	348·7±40·5	36·0	7·6	305·1	−150·2
1914·0	0·0089±0·0019	203·4±11·0	35·6	8·1	159·7	+ 66·5
1915·3	0·0094±0·0027	203·8±24·1	35·0	8·8	160·0	+ 70·1
1916·1	0·0036±0·0013	162·1±31·6	34·6	+ 9·3	118·2	+ 30·3
1917·0	0·0046±0·0010	147·3±18·9	34·1	+ 9·8	103·4	+ 17·9
1918·1	0·0068±0·0016	150·8±21·7	33·6	10·5	106·7	+ 24·0
1919·1	0·0107±0·0020	67·5± 8·7	33·1	11·0	23·4	− 56·9
1919·1	0·0057±0·0013	177·0±15·2	33·1	11·0	132·9	+ 52·7
1920·1	0·0032±0·0022	8·8±36·5	32·6	+11·6	324·6	−113·2
1920·1	0·0034±0·0016	180·0±30·2	32·5	+11·6	135·9	+ 58·3
1921·1	0·0077±0·0013	169·6±11·3	32·1	12·2	125·3	+ 50·2
1921·1	0·0081±0·0019	11·4±15·0	32·0	12·2	327·2	−107·9
1922·1	0·0068±0·0017	203·5±12·6	31·5	12·8	159·2	+ 86·7
1923·3	0·0097±0·0036	5·9±20·8	31·0	+13·4	321·5	−108·1

The values of Q given in column three of Table V. are measured from the ascending node of the satellite's orbit on the Earth's equator. If from these values we subtract ψ (see p. 285) and $\Delta Q=(T-1900)\Delta\theta \cos(180-\gamma)$, we will obtain $Q_F=Q-\psi-\Delta Q$, the longitude of periastron referred to a fixed point in the satellite's orbit, its node on Neptune's equator at 1900·0. The values of Q_F are given in column six of Table V.

Giving to each value of Q_F a weight in accordance with its probable error, weight unity corresponding to a probable error of 31°·6, we obtain from the 25 determinations by Barnard at Yerkes (the largest number of determinations by a single observer at a single instrument):

$$Q_F=155°·3\pm5°·3-(1°·54\pm0·69)(T-1910·0).$$

Similarly from the 8 determinations by A. Hall, Jr., at Washington, 1909 to 1923, we obtain

$$Q_F=11°·1\pm7°·8-(2°·38\pm1°·83)(T-1915·0).$$

While the difference between the values of Q_F from these two series at 1915·0, 148° and 11°, is many times its probable error, 10°, indicating a large systematic difference between the results from these two observers, the difference between the annual motions of Q_F as determined by them is less than half of its probable error.

A comparison of the values of Q_F obtained by the other observers, with the results from the observations of Barnard and A. Hall, Jr., respectively, shows that all the Washington results agree fairly well among themselves and with the results from the visual observations

at the Lick Observatory. These two series include 32 determinations, with probable errors less than 50°, from 1874 to 1923, and yield for the value of Q_F:

$$Q_F = 53^\circ\cdot4 \pm 4^\circ\cdot7 - (2^\circ\cdot79 \pm 0^\circ\cdot36)(T - 1900\cdot0).$$

Finally, a solution including all the values of Table V, except the seven whose probable errors exceed 50°, gives

$$Q_F = 129^\circ\cdot0 \pm 6^\circ\cdot5 - (2^\circ\cdot55 \pm 0^\circ\cdot57)(T - 1900\cdot0).$$

Reducing to 1900·0 the first expressions obtained, we have:

I.	$Q_F = 170^\circ\cdot7 - 1^\circ\cdot54(T-1900\cdot0)$,	25	determinations by Barnard at Yerkes;
II.	$Q_F = 46\cdot8 - 2\cdot38(T-1900\cdot0)$,	8	„ by A. Hall, Jr., at Wash.;
III.	$Q_F = 53\cdot4 - 2\cdot79(T-1900\cdot0)$,	32	„ at Washington and Lick;
IV.	$Q_F = 129\cdot0 - 2\cdot55(T-1900\cdot0)$,	73	„ at various places.

The residuals from the last solution are given in column seven of Table V.

The above expressions for the longitude of periastron indicate an annual motion of the periastron between $-1^\circ\cdot5$ and $-3^\circ\cdot0$, a value considerably larger than one would be led to expect from the determination of the annual motion of the node, $\delta\theta = +0^\circ\cdot6$, and they also show an exceedingly large range in the values of the longitude of periastron itself for the mean epoch of the observations. These results indicate that the existence of an ellipticity in the satellite's orbit is extremely uncertain.

Further details will be published in *Astronomical Papers of the American Ephemeris*, vol. ix., part iii.

Note on the Figure and Rotation Period of Neptune.

By J. Jackson, M.A., D.Sc.

The planet Neptune, as seen from the Earth, presents a disc about $2''\cdot5$ in diameter, usually described as bluish or greyish in colour, apparently perfectly circular and without definite surface markings. Photometric observations of the planet made by Maxwell Hall * at Jamaica in 1915 indicated variability in a period of 7·835 hours. From a discussion of observations made by Müller in 1884–85, by Baldwin in 1908, by Maxwell Hall in 1915, and at Tartu in 1922–23, Öpik and Liviänder † found two periods of $7^h\cdot7067$ and $7^h\cdot8363$ superposed. The range of the variation was about $0^m\cdot15$ for each of the periods, but, even although the two periods may be attributed to different zones of the planet, further confirmation of these periods would be useful. The

* *M.N.*, **75**, 626.

† *Publ. del' Obs. Astr. de l'Univ. de Tartu* (Dorpat), **35**, No. 7.

preceding paper by Professor Eichelberger and Mr. Newton fixes the position of the axis of rotation, and from the rate of motion of the pole of the satellite's orbit about the pole of the planet's equator, conclusions can be drawn with regard to the figure of the planet and its speed of rotation.

The angular motion $\delta\theta$ of the node of the satellite's orbit on the planet's equator is given * by

$$\delta\theta=\frac{b^2}{a^2}(e-\tfrac{1}{2}\phi)n\cos\gamma,$$

where b is the equatorial radius of the planet, a the semi-major axis of the satellite's orbit, n the sidereal motion of the satellite, γ the constant angle between the planes of the satellite's orbit and the planet's equator, e the ellipticity of the planet, and ϕ the ratio of the centrifugal force to gravity at the planet's equator. The quantity $(e-\frac{1}{2}\phi)$ is deduced from the quantity $\frac{3\,(C-A)}{2mb^2}$, where m is the mass of the planet, and C and A the moments of inertia about polar and equatorial diameters. The only assumptions made in this reduction are that the surface of the planet is one of equilibrium corresponding to the speed of rotation and that the planet's material is stratified. No assumption is made as to the change of density with depth.

Putting $a/b=13{\cdot}1$, and using the values of $\delta\theta$, n, γ given by Eichelberger and Newton, we find

$$e-\tfrac{1}{2}\phi=1/199{\cdot}1.$$

We now have to adopt a ratio for $(e-\frac{1}{2}\phi)/e$ or $1-\frac{1}{2}\frac{\phi}{e}$. For a homogeneous body $\phi=\frac{4}{5}e$, so that this ratio is 0·6. For the earth it is 0·48, for Jupiter 0·35, for Saturn 0·21. For a body with all its mass concentrated at the centre the value of the ratio is 0. The figures indicate that the order Earth, Jupiter, Saturn is one of increasing central condensation; and this is supported by the following table, giving values of $1-\frac{1}{2}\frac{\phi}{e}$ and the density:—

	$1-\frac{1}{2}\frac{\phi}{e}$.	Density.
Earth . . .	0·48	1·00
Jupiter . .	0·35	·25
Saturn . . .	0·21	·13
Neptune . .	?	·22

It seems reasonable to assume for Neptune $1-\frac{1}{2}\frac{\phi}{e}=\cdot33$. We then deduce $e=1/65{\cdot}7$. As Neptune has a diameter of only 2″·5, the difference between the equatorial and polar diameters would be only 0″·04.

Once e has been found, we deduce $\phi=1{\cdot}34e$ on the above assumptions.

* See Tisserand, *Mécanique Céleste*, iv. p. 143.

If T be the period of rotation, ρ the mean density of the planet, and T′, ρ', ϕ' the corresponding quantities for the earth, then

$$T = T'\sqrt{\frac{\rho'\phi'}{\rho\phi}}.$$

We have $T' = 24^h$, $\rho/\rho' = \cdot 222$, $\phi' = \frac{1}{288\cdot 4}$, $\phi = \frac{1}{49\cdot 0}$, whence T = 19·1 hrs.

The above values are subject to considerable uncertainty. The probable error of $\delta\theta$ is 11 per cent., and of a/b perhaps 5 per cent. This gives $e - \frac{1}{2}\phi$, with a probable error of about 15 per cent. The use of ·30 as the value of $1 - \frac{1}{2}\frac{\phi}{e}$ gives $e = \frac{1}{59\cdot 8}$ and $\phi = 1\cdot 4e = \frac{1}{42\cdot 7}$, T = 17·8 hours. From these figures we might expect the error of T not to exceed 15 or 20 per cent.

The period of rotation obtained above differs widely from that based on the photometric observations. The value of 7·835 hours seems too small. For if we assume this value we can compute $\phi = 1/6\cdot 5$, and $e = 1/12\cdot 2$, and $1 - \frac{1}{2}\frac{\phi}{e} = \cdot 061$. The data for the other planets hardly supports a value of this kind. Also so large an oblateness might be observed at the telescope under favourable conditions. A way out of the difficulty may be found in assuming that the rotation period is two or three times that derived from the photometric observations, a result which could be attributed to darker or brighter areas on the planet at intervals of approximately 180° or 120° in longitude.

A period of 19·1 hours leads to an equatorial velocity of 2·5 km/sec., or a difference in the radial velocity of 5·0 km./sec. between opposite limbs. With a disc of 2″·5 in diameter and a total brightness equal to that of a star of the eighth magnitude, it seems doubtful if our present means are sufficient for a determination of the rotation period by the Doppler effect.

Note on the Mass of Venus.

By J. K. Fotheringham, D.Litt.

In his provisional theory Leverrier used for the mass of Venus, expressed in terms of the mass of the Sun, the value $\frac{1}{401847}$. In his *Astronomical Constants* (1895) Newcomb treats this value as unity, and expresses corrections as a fraction of this value. The value used by Leverrier in his tables of the Sun implies a correction of +·004 to this value, while Newcomb himself deduces from observations of the inequalities produced by Venus in the position of the Sun and Mercury respectively the corrections −·0118±·0023 and −·0121±·0034.* Later

* *Astronomical Constants*, p. 101. Newcomb gives the mean errors, which I have reduced to probable errors.

in the same work, after discussing the supposed effect of Venus on the secular variations of the elements of the inner planets, he arrives at his final correction of −·0151, giving Venus a mass of $\frac{1}{408000}$.* An opinion has since then gained ground that Leverrier's value was nearer to the truth than Newcomb's. Dr. Cowell, in *M.N.*, **66** (1906), 302–7, found from Greenwich observations of the Sun, 1864–1900, a correction of +·003 to the value used in Leverrier's tables of the Sun, *i.e.* of +·007 to the value used in Leverrier's provisional theory, while Dr. Ross in *Astronomical Papers of the American Ephemeris*, **9**, ii. (1917) p. 261, deduced from observations of Mars a correction of +0·0112±·0058 to Newcomb's final value, which is equivalent to a correction of −0·00405 ±0·0057 to the value used in Leverrier's provisional theory. Founding on these two papers, Sir Frank Dyson and Dr. Crommelin have, in a paper on Greenwich Observations of the Moon, *M.N.*, **83** (1923), 366, applied to Dr. Brown's value of the great Venus term in the motion of the Moon a correction of +1/80th of the value of that term. This is equivalent to a correction of +·0123 to Newcomb's final value of the mass of Venus or of −·0028 to the value used in Leverrier's provisional theory.† In the present paper I endeavour to show that these investigators have exaggerated the defects of Newcomb's determinations, and that the mass of Venus is not much greater than Newcomb supposed.

Dr. Cowell's determination is based upon the Greenwich meridian observations of the Sun from 1864 to 1900, which he analyses for the two largest inequalities produced by Venus in the longitude of the Sun. Expressing his results as corrections to Leverrier's provisional value for the mass of Venus, we find that he gets

from the term in V−E +·010,
and from the term in 2(V−E) +·004,
yielding a mean correction of +·007.

Newcomb, in his *Astronomical Constants*, p. 22, had included the same observations as far as 1892 in his discussion of the observed mass of Venus, using all the inequalities, and had solved these observations for groups containing generally four years each. If Newcomb's last six groups, containing the observations from 1867 to 1892, are combined in a separate solution, they yield a correction of +·006±·0055, where the probable error is deduced from the mutual discordances of the different groups. Dr. Cowell noticed this accord between his conclusion and Newcomb's from the particular series of observations, and considered that Newcomb's final result was due to his swamping these good Greenwich observations by introducing additional material of decidedly inferior value. This additional material consists partly of older observations made at Greenwich and elsewhere, and partly of contemporary observations made at other observatories. Dr. Cowell's contention implies that the particular series of Greenwich observations

* *Astronomical Constants*, p. 175.

† Dr. Jones, *General Astronomy* (1922), p. 232, refers to Dr. Ross's value as the most probable value of the mass of Venus.

was not only better than all older observations and than contemporary observations made elsewhere, but so much better that we can afford to ignore all other meridian observations. It is clear from the weights that Newcomb attached to these observations that he would have admitted their superiority. The question is the degree of superiority.

It is difficult to find any cause of systematic error in the Venus observations, because, so far as we can see, all possible instrumental errors would be assigned to other terms of different period. Dr. Cowell regards this as a reason for believing that no large error is possible in the result deduced from the observations discussed by him, but the argument would seem to apply with equal force to the observations not discussed by him. Dr. Cowell suggests, but does not prove, that large discordances may exist in the older observations, and remarks that the older observations of the Sun and Moon had not been re-reduced with the same care that had lately been bestowed on several of the earlier star catalogues. So far as re-reduction is concerned, Newcomb's description on pp. 1, 2 of his *Astronomical Constants* is impressive. As regards discordances, Newcomb professes to have weighted small groups of not more than six or eight observations, according to the discordances of the residuals found in the results of each observer. If we pass from discordances of small groups to discordances of larger groups, for which Newcomb has published separate solutions, it is possible to judge the extent of the discordances for ourselves. On p. 22 Newcomb divides the Greenwich observations into twenty-nine groups. Combining the first five of these into one super-group, and the remainder into four super-groups containing six groups each, the super-groups give the following solutions. I retain Newcomb's weight for each group, and find the probable error from the discordances of the individual groups.

Years.	Correction to Mass.	Combined Weight (Newcomb).	Weight deduced from Discordances.
1750–92	$-\cdot031\pm\cdot0023$	53	169
1793–1818	$-\cdot042\pm\cdot018$	29	3
1819–42	$+\cdot006\pm\cdot012$	41	6
1843–66	$-\cdot021\pm\cdot0071$	50	18
1867–92	$+\cdot006\pm\cdot0055$	53	28

It will be observed that the discordances of the super-groups are greater than the probable errors, determined from internal discordances, would lead us to expect, and it will be seen that the super-group examined by Dr. Cowell gives an extreme value in one direction. On the other hand, Dr. Cowell appears to be justified in supposing that Newcomb had attached excessive weight to the earlier observations, though, in this case, the first super-group with its highly consistent results is an exception. If we solve with the weights given in the last column, the resultant correction to the mass of Venus is $-\cdot025\pm\cdot0045$. If we solve with Newcomb's weights, it becomes $-\cdot015\pm\cdot0063$.

I next compare the last super-group with super-groups made from

contemporary observations at other observatories, and tabulate as follows :—

Super-Group.		Correction to Mass.	Combined Weight (Newcomb).	Weight deduced from Discordances.
Greenwich,	1867–92	+·006±·0055	53	71
Paris,	1866–89	+·017±·011	32	21
Washington,	1866–91	−·011±·0065	42	54
Oxford,	1869–87	+·029±·022	24	5
Cape of Good Hope,	1884–90	−·026	12	..
Strassburg,	1883–88	−·014	12	..

If we assign weight 12 with Newcomb to each of the two last observatories, and take the weights for other observatories from the last column, the resultant correction from the six observatories is −·001±·0088. The Greenwich observations cannot be said to be discordant from the other observations within the same range of years, but once more the discordances between the different observatories are greater than we should expect from the internal discordances of each. Neither comparison seems to show that the Greenwich observations of 1864–1900 are entitled to outweigh all other observations. In Newcomb's final solution from observations of the Sun, the Greenwich series 1867–92 receives approximately a weight of 3 out of a total weight of 34. The above study suggests that it may be entitled to greater weight than Newcomb has given it, but certainly not to twice the weight assigned to it. Yet if we were to double its weight, we should only change the final result from observations of the Sun from −·0118 to −·0108. I do not propose to make this correction.

Dr. Ross's examination of the observations of Mars is excellent, so far as I am entitled to express an opinion. For the reasons stated by Dr. Ross, we should expect a stronger result from the observations of Mars than from those of the Sun. Dr. Ross's final solution is based on five groups of observations, and here again we find that the discordance of the groups is greater than we should expect from their individual probable errors. The two last groups, covering the years 1847–75 and 1877–1912 respectively, are extremely discordant from each other. In these observations, as in those of the Sun, it would be unwise to base conclusions on any group to the exclusion of others.

The final correction from observations of Mars is as stated above, −0·00405±0·0057. Dr. Ross compares this with Newcomb's results from the Sun and Mercury, but is guilty of two curious oversights. In the first place, he mistakes Newcomb's mean errors for probable errors, and thus underrates the strength of his conclusions ; and secondly, he assumes that Newcomb's probable errors are obtained by the customary weight method, while his own are computed from the discordances of five independent values. He remarks that his probable error would have been reduced to ±·0020, if he had used the customary weight method. Now a reference to Newcomb's determination of the mass of Venus from Sun observations on p. 25 of *Astronomical Constants* will show that his probable error is computed from the discordances of ten independent values, for whereas Dr. Ross has solved separately for different epochs,

Newcomb has solved separately for different observatories. I conclude, therefore, that Newcomb's result from the Sun is strictly comparable with Dr. Ross's result from Mars.

Collecting the different solutions, which are in good agreement, we have :

From observations of the Sun $-\cdot0118\pm\cdot0023$,
,, ,, Mercury $-\cdot0121\pm\cdot0034$,
,, ,, Mars $-\cdot00405\pm\cdot0057$,
From Sun, Mercury, and Mars combined $-\cdot0111\pm\cdot0018$.

The corresponding mass of Venus, taking the mass of the Sun as unity, is

$$\frac{1}{406358\pm723}.$$

University Observatory, Oxford :
1926 *March* 11.

The Velocities of Ions under Radiation Pressure in a Stellar Atmosphere, and their Effect in the Ultra-Violet Continuous Spectrum. (*Second Paper.*) By Martin C. Johnson, M.A., M.Sc.

I. Radiative and Electrostatic Forces.

1. *Introduction.*—In a previous communication * an investigation was made of the effects of selective radiation-pressure in driving hydrogen and helium up through the outer chromosphere of a very hot star, and some consequences were discussed which might lead to the high capture velocities of electrons implied by the ultra-violet continuous spectra of certain stars and nebulæ. The present paper is an extension of the investigation to stars ranging from solar to O-star temperatures, in order to find at what point in the stellar sequence the various effects begin, and it is also an attempt to profit by criticisms of Paper A.

2. *Theory of an Assemblage of Ions exposed to Radiation.*—The theory of the upper parts of a stellar atmosphere differs from that applicable to an ordinary gaseous assemblage in several ways, of which the most important are the following :—

(*a*) Whereas gravitational equilibrium would distribute the constituents of an approximately isothermal atmosphere on Dalton's law solely according to their molecular weights, radiation-pressure will not be selective according to mass, and may, as shown in Paper A, outweigh gravitational attraction under favourable conditions.

(*b*) The fact that a large proportion of particles are charged introduces more than one complication. Any separation of kinds of charged particles due to gravitation through their differing masses, or to radiation-pressure through their differing absorbing powers, will mean a tendency of various zones to develop a space charge. This tendency will

* *M.N.*, **85**, 813. (Subsequently referred to as Paper A.)

readjust the separations as they occur. A particular case of this is the known effect of a positive charge on a star in preventing the outflow of light electrons natural to an atmosphere in gravitational equilibrium.

(*c*) In the upper parts of a star's atmosphere thermodynamic equilibrium considerations may have to be seriously questioned owing, firstly, to breakdown of Maxwellian velocities in the enormous free paths between atomic collisions, and, secondly, to the nature of the ionising agency. This agency is not the temperature of the medium and of local origin, but is a unidirectional photospheric stream of radiation penetrating a region which itself may be too rarefied to obey the temperature laws of ordinary kinetic theory.

3. *Conditions of Equilibrium in Gravitational, Electrostatic, and Radiative Fields.*—Since throughout any stellar atmosphere density changes will be much more important than temperature changes, let us start by correcting the usual hydrostatic equation of isothermal equilibrium by some of the preceding considerations.

If p and p' are the partial pressures of two constituents of the atmosphere, whose separate particles are of mass m and m' respectively, and if κ is Boltzmann's constant and T the temperature, and if E is the potential per unit mass of any field of force,

$$p=p_0e^{-m\mathrm{E}/\kappa\mathrm{T}},$$
$$p'=p'_0e^{-m'\mathrm{E}/\kappa\mathrm{T}},$$

where p_0, p_0' are the pressures at some datum height. In the commonest case the field is gravitational and

$$\mathrm{E}=gx,$$

where x is a height above the surface of a star or planet and g is the gravitational acceleration.

Milne has treated * the case of electrostatic potential due to the tendency of the electrons to diffuse upwards because of their small mass, and has shown that the purely gravitational expression for the potential is not sufficient, until at a pressure below 10^{-34} atmosphere the ions are too far apart to exert any appreciable influence on each other.

Here we have to consider the electrostatic potential V, and also the fact that gravitational acceleration may be reduced or even reversed in direction by a selective radiation-pressure giving rise to an acceleration R. Hence,

$$p=p_0e^{-[m(g-\mathrm{R})x+e\mathrm{V}]/\kappa\mathrm{T}},$$
$$p'=p'_0e^{-[m'(g-\mathrm{R}')x-e\mathrm{V}]/\kappa\mathrm{T}},$$

where two types of particle have charges e of opposite sign.

Now Pannekoek and Rosseland † have shown that when equilibrium is reached between the gravitational diffusion of electrons and the electrostatic field set up opposing that motion, if E_1 be the part of the potential E due to the electrostatic charges and E_2 the gravitational part,

$$e\mathrm{E}_1=-\tfrac{1}{2}(m-m')\mathrm{E}_2.$$

* *Proc. Camb. Phil. Soc.*, **22**, 493, 1925.

† See discussion by Milne, *loc. cit.*

Adapting this to the present case, in equilibrium

$$eV=-\tfrac{1}{2}x[m(g-R)-m'(g-R')] \quad . \quad . \quad . \quad (1)$$

The physical interpretation of this equation is that not only will a field be set up opposing the gravitational separation of heavy and light ions, but that similar fields will oppose the separations due to differential radiation-pressure. The action of the field in the two cases will, however, be different. In the purely gravitational case the balance between the separating field and the field opposing the separation constitutes a mechanical equilibrium, and the combined system does not move in any one direction; in the radiative case there may or may not be mechanical equilibrium, and this balance of fields may be superposed on a steady unidirectional motion of both kinds of ions. This latter will occur if the radiation-pressure on the heavy ions is greater than gravitation, or if the radiation-pressure on the electrons is so much greater than gravitation as to support not only electrons but heavy ions too.

The present problem is to investigate the values of R and R′ in equation (1), for various elements in stars of various gravitational properties and temperatures.

4. *Gravitational Accelerations on Giants and Dwarfs and early and late Stars.*—Variations in g from star to star will, on the whole, be the least important of the properties determining the equilibrium. But in a few cases they may be critical, and it will be well to tabulate some normal and extreme values.

If G is the constant of gravitation, ρ the mean density, and r the radius of a star,

$$g=\tfrac{4}{3}\pi r\rho G.$$

If the gravitational acceleration at the surface of the sun is taken as equal to $2{\cdot}7\times10^4$ cm. per sec.^{2}, use may be made of comparisons of densities and diameters and densities and masses,* from which the following table is calculated:—

TABLE I.

Values of g at Stellar Surfaces.

Type.	Temperature.	Mass × ⊙.	ρ.	$g/10^4$ cm. sec.$^{-2}$.
G_0 dwarf	6,000	1·0	·68	1·7
G_0 giant	6,000	2·6	·0025	·057
G_0 giant	6,000	10·0	·00002	·0032
A_0 dwarf	10,000	6·0	·36	2·0
A_0 giant	10,000	7·0	·16	1·2
A_0 giant	10,000	10·0	·008	·19
B_2	15,000	10·0	·07	·81
B_2 (Plaskett's star)	15,000	86·0	·01	·43
O giant	30,000	50·0	·007	·30
White dwarf	..	..	50,000	3000

* Seares, *Ap. J.*, **55**, 165, 1922; Plaskett, *M.N.*, **82**, 447, 1922; Plaskett, *Pub. Dom. Ast. Obs.*, 1924.

5. *Radiative Accelerations of various Elements at various Star Temperatures.*—(i) It is known * that the upward acceleration of an atom due to the absorption of monochromatic radiation from below is given by

$$R=\frac{1}{c}m\frac{\kappa}{\rho}F_\lambda\tau r/m \quad . \quad . \quad . \quad . \quad (2)$$

where c is the velocity of radiation, m the atomic mass, κ/ρ the mass absorption coefficient, F_λ the intensity of radiation available in ergs per sq. cm., τ the fraction of its life that the atom spends in a state capable of absorbing the wave-length λ, and r the residual fraction of photospheric radiation of that wave-length not already absorbed in the reversing layer.

In terms of Planck's equation and the wave-length interval $d\lambda$,

$$F_\lambda=2hc^2d\lambda/\lambda^5(e^{hc/\lambda\kappa T}-1).$$

Now if an atom is capable of existing in n different quantum states, in each of which a different radiation can be absorbed,

$$\tau_n=1/\{A_{n\to1}+\ .\ .\ .+A_{n\to(n-1)}\},$$

where $A_{n\to1}$, etc., are the probabilities of leaving state n for state 1, etc. In Paper A an attempt was made to calculate these probabilities in the case of three possible states, but for $n>3$ the calculations become complex and the error involved in finding probabilities becomes at least as great as the error made in confining attention to cases where only two states are of importance. In this latter type of case,

$$\tau_1/\tau_2=f_1/f_2,$$

where f_1, etc., are the fractional concentrations of atoms in those particular states. This approximation becomes all the more reasonable since calculations of concentration depend on the equations of thermodynamic equilibrium; for a high chromosphere these are subject to the source of error mentioned in 2 (*c*). The justification for using the equations at all under these circumstances must be referred to Paper A, p. 818.

The equations we shall use for calculating f, and hence, by the above approximation, estimating τ, are the Saha-Fowler-Milne equations

$$1-x=\frac{b(T)/b'(T)}{b(T)/b'(T)+\alpha T^{\frac{3}{2}}e^{-\chi_1/\kappa T}}$$

$$f_r=\frac{b'(T)q_re^{-\chi_1-\chi_r/\kappa T}}{b(T)},$$

where $\alpha=0{\cdot}332\sigma/P_e$, $1-x$ is the unionised fraction of atoms at temperature T and partial pressure of electrons P_e, χ_1 is an ionisation potential, κ is Boltzmann's constant, σ is the number of valency electrons in equivalent orbits, and q is the weight of a state defining the partition functions $b(T)$ and $b'(T)$ for normal and ionised states

* Paper A, p. 816 and p. 821

respectively. These equations are amended since Paper A according to Milne's note * on the use of $b'(T)$. f_r is the fraction of the unionised atoms in the r-state.

(ii) The available energies are first calculated for $d\lambda = 2\times10^{-8}$ cm. for various hydrogen and helium series. For reasons given later the temperature quoted in Paper A (40,000°) is not used, but different temperatures and the energy at the first line, as well as at the limit of each series, are considered.

TABLE II.

Values of F_λ *at Limit and First Line of* H *and* He *Series.*

			6000°.	10,000°.	15,000°.	30,000°.
H Lyman	1st line	λ 1,215	$1{\cdot}7\times10^{3}$	$5{\cdot}0\times10^{6}$	$2{\cdot}7\times10^{8}$	$1{\cdot}4\times10^{10}$
	Limit	λ 912	$1{\cdot}1\times10^{1}$	$4{\cdot}5\times10^{5}$	$1{\cdot}0\times10^{8}$	$2{\cdot}0\times10^{10}$
H Balmer		6,564	$5{\cdot}0\times10^{6}$	$2{\cdot}5\times10^{7}$	$5{\cdot}8\times10^{7}$	$2{\cdot}0\times10^{8}$
		3,646	$5{\cdot}0\times10^{6}$	$7{\cdot}0\times10^{7}$	$2{\cdot}8\times10^{8}$	$1{\cdot}4\times10^{9}$
He normal		584	$2{\cdot}0\times10^{-5}$	$5{\cdot}0\times10^{2}$	$1{\cdot}5\times10^{6}$	$9{\cdot}0\times10^{9}$
		500	$1{\cdot}1\times10^{-7}$	$5{\cdot}4\times10^{1}$	$4{\cdot}2\times10^{5}$	$5{\cdot}7\times10^{9}$
He excited			5×10^{6}	3×10^{7}	8×10^{7}	3×10^{8}
(Mean of doublet and singlet series)			5×10^{6}	7×10^{7}	3×10^{8}	1×10^{9}
He^+ normal		304	1×10^{-20}	1×10^{-6}	1×10^{1}	9×10^{7}
		228	3×10^{-37}	6×10^{-16}	6×10^{-6}	$1{\cdot}5\times10^{5}$
He^+		1,640	$6{\cdot}0\times10^{4}$	$3{\cdot}3\times10^{7}$	$6{\cdot}0\times10^{8}$	$1{\cdot}2\times10^{10}$
		911	$1{\cdot}1\times10^{1}$	$4{\cdot}5\times10^{5}$	$1{\cdot}0\times10^{8}$	$2{\cdot}0\times10^{10}$
He^+		4,686	$6{\cdot}6\times10^{6}$	$5{\cdot}3\times10^{7}$	$1{\cdot}6\times10^{8}$	$6{\cdot}0\times10^{8}$
		2,050	$4{\cdot}1\times10^{5}$	$5{\cdot}5\times10^{7}$	$5{\cdot}5\times10^{8}$	$6{\cdot}6\times10^{9}$
He^+		10,100	$2{\cdot}6\times10^{6}$	$7{\cdot}6\times10^{6}$	$1{\cdot}4\times10^{7}$	$3{\cdot}5\times10^{7}$
		3,644	$5{\cdot}0\times10^{6}$	$7{\cdot}0\times10^{7}$	$2{\cdot}8\times10^{8}$	$1{\cdot}4\times10^{9}$

The proportions of H and He atoms in normal, excited, and ionised states are then calculated for the same temperatures, and for $P_e = 10^{-4}$ atmosphere, the typical reversing layer pressure. The following data are used :—

	$b(T)$.	$b'(T)$.	σ.	χ_1 volts.	$\chi_1-\chi_r$.	q_r.
H	2	1	1	13·54	10·16	6
He	1	2	2	24·7	21·2	2
He^+	2	1	1	54·2		

It is clear from Tables II. and III. that there is an enormous preponderance of time spent in the normal state if an atom is not ionised, and that normal He^+ radiations are extremely weak in all but the hottest stars; hence the radiation-pressure which is due to absorption by He^+ will not be important except in the two highest frequency He^+ series and at 30,000°. At this temperature the unionised fraction of He is

* *Phil. Mag.*, **50**, 547, 1925.

TABLE III.

Proportions of Atoms in Different States, for H *and* He.

	Temperature.	x.	$1-x$.	f_r.	$f_r(1-x)$.
H	6,000°	2×10^{-5}	1	8×10^{-9}	8×10^{-9}
	10,000	0·7	0·3	$2\cdot2\times10^{-5}$	7×10^{-6}
	15,000	1	$0\cdot8\times10^{-3}$	$1\cdot1\times10^{-3}$	10^{-6}
	30,000	1	$0\cdot7\times10^{-6}$	$6\cdot1\times10^{-2}$	$4\cdot3\times10^{-6}$
He	6,000	10^{-14}	1	$6\cdot2\times10^{-18}$	$6\cdot2\times10^{-18}$
	10,000	3×10^{-5}	1	$5\cdot6\times10^{-11}$	$5\cdot6\times10^{-11}$
	15,000	0·68	0·32	$4\cdot8\times10^{-7}$	$1\cdot5\times10^{-7}$
	30,000	1	7×10^{-6}	$1\cdot1\times10^{-3}$	$7\cdot7\times10^{-9}$

less than 10^{-5}, so we may neglect it and start again with the ionisation potential of He^+. The use of the same equations as before shows the distribution in the normal and next to normal states to be as follows :—

		x.	$1-x$.	f_r.	$fr(1-x)$.
He^+	30,000°	·17	·83	3×10^{-7}	$2\cdot5\times10^{-7}$

(iii) Before drawing any conclusion from these tables it is necessary to consider in more detail the remaining factor r in equation (2).

Milne * has discussed the dependence of radiation-pressure on r from the point of view of equilibrium between gravitational and radiative accelerations : in a star with a strong absorption line the upper chromosphere is shielded from the full impact of the photospheric radiation, and atoms which might have been supported against gravity by absorbing that particular frequency will fall down to a lower level. Milne has remarked that in the hotter stars it is improbable that r should actually reach the very low value required to prevent calcium being driven off the star by radiation-pressure. Here we are only concerned with estimating a reasonable mean value for the types of star we have been discussing. r is taken as 0·1 in Milne's work on the H and K lines in the sun, and we will adopt this value for all lines in the 6000° star. Since the H and He lines are especially prominent in the reversing layers of A and B stars, this value must be reduced to ·01, say, for these lines in 10,000° and 15,000° stars. For the 30,000° star we will adopt the value unity for r, for the reasons specified in Paper A, p. 817.

(iv) The values of R for H and He in equation (1) will now be calculated from Tables II. and III., using the aforementioned approximate method of estimating the value of τ. The mean estimates just discussed for the value of r will be used, and κ/ρ will be taken as 10^8, since all theoretical and experimental determinations of monochromatic absorption coefficients are distributed round this figure.

In all cases where $R<g$ the maximum value of R only will be tabulated. That is to say, the energy available will be taken from the first line or the series limit, whichever happens at that temperature

* *M.N.*, **84**, 354, 1924.

distribution to have the most energy, and only that quantum state will be considered in which the longest time is spent. Secondary or minimum values will only be given when $R>g$. It is at once seen that only the values of g for giant stars are required.

TABLE IV.

Relation of Radiative to Gravitational Acceleration.

Temperature.	Element.	R/g.	Series.
6,000°	H	$0{\cdot}5/32$	Lyman.
	He	$10^{-9}/32$	Normal.
10,000	H	$45/1{\cdot}9\times10^{3}$	Lyman.
	He	$10^{-2}/1{\cdot}9\times10^{3}$	Normal.
15,000	H	$6{\cdot}5/3{\cdot}8\times10^{3}$	Lyman.
	He	$14/3{\cdot}8\times10^{3}$	Normal.
30,000	H	$40/3\times10^{3}$	Lyman.
	He	$190/3\times10^{3}$	Normal.
	He	$2{\cdot}5\times10^{5}/3\times10^{3}$	1st line of ionised normal.
	He	$370/3\times10^{3}$	Limit of ionised normal.
	He	$15/3\times10^{3}$	1st line of ionised Lyman.

Three conclusions are apparent from this table : first, that in no case is radiative acceleration greater than gravitational, except for He at 30,000°. Secondly, radiation-pressure on an element may have more than one maximum and turning-point in the stellar sequence, owing to the number of combining conditions which go to make up the absorbing power. Thirdly, radiation-pressure in all cases is only effective through normal state absorption.

(v) Milne has already indicated the probability of radiation-pressure overbalancing gravitation in the hotter stars in the case of calcium. We will see if any conclusions with regard to C, Si, O, N can be added to the cases of Ca and He, as these elements are prominent in the spectra of the hotter stars.

The lines associated with those of He^{+} and attributed to C^{+}, C^{++}, and C^{+++} are not principal series lines, and we may assume that the important radiation-pressure is derived, as in the case of H and He, from principal series which are not seen in the spectrograms. The limits of these can be found from the ionisation potentials. If λ is the wave-length at a principal series limit and λ_1 is the wave-length of the first line,

$$1/\lambda = N/1^2 - N/\infty$$
$$1/\lambda_1 = N/1^2 - N/2^2,$$

where N=ionisation potential in volts/$1{\cdot}234\times10^{-4}$.

C^{+}(I.P. 24·3 volts) will have much the same small radiation-pressure as He(I.P. 24·5), but C^{++}(I.P. 45) will contribute radiation-pressure due to absorption of a series which can be calculated to have a first

line at λ370 A., and a limit at λ277 A. Taking the Fowler and Milne * figures for the concentration of C^{++} atoms at the maximum of the visible subordinate lines of this stage of ionisation, and using as before the Planck equation for the intensity of the radiation of this series at the temperature of maximum (26,600°), the maximum radiative acceleration of carbon is found to be $1{\cdot}5\times10^5$ cm. per sec.^{2}, and the fraction R/g corresponding to Table IV. is 50. Carbon will then be subject to an acceleration outwards of the same order of magnitude as that affecting helium.

Another element associated with the spectra of the hotter stars is silicon, of which subordinate Si^{+++} lines are seen. The ionisation potential for this stage is 44·9 volts, and the lines also have a maximum at 26,600°. Fowler and Milne show that the concentration of normal atoms at this temperature in this stage is slightly larger than in the case of C^{++}, so we may expect radiation-pressure of at least as great effectiveness due to absorption of the principal series.

Other elements associated with these include oxygen and nitrogen. The ionisation potentials of O^{++} and N^{++} are known to be between 48 and 53 volts, but since the sequences of the potentials are not completely known we cannot be certain that each stage is separated from the preceding stage of ionisation sufficiently to apply the equations rigorously. We must recognise, however, that the potentials indicate a large probability that these two elements also have the fraction R/g considerably greater than unity under circumstances fulfilled in many O-type stars.

(vi) The possibility of radiation-pressure counterbalancing gravity for free electrons must now be considered.

In Paper A this radiation-pressure was calculated from the mass-scattering coefficient discussed in detail in *M.N.*, **85**, 56, 1924. This coefficient, however, was derived from experimental measurements in which a gas density was involved, and becomes misleading when the pressure on an isolated electron apart from the atoms of a gas is to be considered. This requires that the electron radiation-pressure be recalculated on the basis provided by the charge and mass data above. The " cross-section for scattering " of a free electron will be accordingly taken from the Thomson equation as equal to

$$\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2,$$

and the acceleration due to radiation-pressure as equal to

$$\frac{1}{cm}\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2\sigma T^4$$

$$=1{\cdot}37\times10^{-12}T^4,$$

where e is the electronic charge, m the electronic mass, c the velocity of radiation, σ is Stefan's constant, and T the temperature.

* *M.N.*, **84**, 499, 1924.

The numerical values found are as follows :—

TABLE V.

Radiative and Gravitational Acceleration of Electrons.

T.	6000°.	10,000°.	15,000°.	30,000°.
R′ cm./sec.^{2}	$0{\cdot}17\times10^4$	$1{\cdot}37\times10^4$	$6{\cdot}8\times10^4$	111×10^4
	Dwarf 0·1	0·7		
R′/g	Giant 3·0	1·1	10	370
	Giant 53·0	7·2		

Except for dwarfs, radiative acceleration due to scattering will always be greater than gravitational acceleration for electrons.

Before leaving this question the possibilities of other forms of radiation-pressure on electrons must be briefly mentioned.

Firstly, what radiation-pressure on electrons is to be expected from photo-electric ionisation of atoms? If the frequency of a principal series limit is absorbed, an electron is removed from the atom. If a higher frequency is absorbed, the surplus energy increases the velocity with which the electron is ejected. If the ejection direction bears any definite relation to the direction of the radiation-stream there may be additional radiation-pressure on electrons in photo-electric ionisation at high frequencies. Recent experimental investigation * and theoretical studies agreeing with experiment † are not easy to apply to the stellar problem, because they concern beams of radiation stopped down to narrow pencils. The small asymmetry of photo-electric emission which they discuss will, however, be even less noticeable in a less homogeneous stream, and we may conclude that the directional properties of photo-electric ejection velocities differ from random so very little as to increase but slightly the net radiation-pressure on electrons calculated on scattering.

Secondly, a new type of scattering near an absorption frequency has recently been discussed by Kramers and Heisenberg.‡ The radiation-pressure associated with this scattering makes it necessary to look upon the pressures of Table V. as a lower limit in each case. Since only in the dwarf stage and the cooler stars is the ratio R′/g as low as unity, and later paragraphs of this paper are concerned chiefly with the problems of the hotter stars, the Kramers-Heisenberg scattering need not be considered as critical here.

6. *The Movements allowed by the Electrostatic Field.*—It is now possible to reconsider the breakdown of mechanical equilibrium suggested in 3, in the light of Tables IV. and V. In dwarf stars at 6000° and 10,000° neither electrons nor atoms are likely to have their gravitational equilibrium broken down by radiation-pressure. The fact of differential radiation-pressure, *i.e.* the fact that R′/g>0·1 and <1·0 for dwarfs, while R/g for H is of the order 10^{-2} and for He much smaller still, will merely produce an effect analogous to that already studied by

* Bothe, *Zeitschr. f. Ph.*, **26**, 59, 1924.
† Mie, *Zeitschr. f. Ph.*, **33**, 33, 1925.
‡ *Zeitschr. f. Ph.*, **31**, 681, 1925.

Pannekoek, Rosseland, and Milne. In their problem the different weight of electrons and atoms is prevented from producing a full Dalton separation by the electrostatic field set up by the slightest separation; in the present case differences in the potential are due to the differential radiation-pressure, but are of the same order of magnitude as the differences in the gravitational field of electrons and atoms. In dwarfs, then, no mass motions due to radiation-pressures will be expected, except by local temporary increases of radiative flux such as possibly give rise to solar prominences.

The only case where important radiation-pressure on positive ions will occur is that of the 25–30,000° star, when R/g can be of the order 10^2 and R'/g of the order 4×10^2. Here the differential radiation-pressure will again be unable to separate the positive aggregate from a negative aggregate, which would otherwise leave the star faster than the positive, and the two groups of particles will act together with only the slight separation sufficient to set up the counterbalancing field. But in this case the condition will not be one of equilibrium in the field but of motion. The action of the field is to ensure that for every positive ion of n-fold charge that moves outwards n electrons shall also move out, there being nothing in the equations of paragraph 3 to stop this streaming out of positive and negative ions together.

It is necessary to inquire whether any extreme value of R'/g may not take the place of large R/g and produce the outstreaming in one of the cases where it cannot be produced on the grounds of the value of R/g alone, *e.g.* in the case of H in the hottest star.

For H, $m/m'=1845$, hence radiative acceleration of an electron must be nearly 2×10^3 times gravitational acceleration to enable the electron to drag the positive ion with it. This condition is seen from Table V. not to be fulfilled even at 30,000°. For He, $m/m'=4\times1845$, and Table IV. shows the impossibility of electronic radiation-pressure effecting here what is only possible through quantum absorption. In the case of the heavier elements the mass ratio is greater still, and it seems necessary to conclude that radiation-pressure on electrons can never do more than effect the slight drag on the positive ions, which appears even in the non-radiative kinetic theory of the escape of electrons from a star by collision impulses.

Hence a theory of radiation-pressure and electrostatic forces between negative and positive ions constructs a picture of all but the hottest stellar atmospheres, in equilibrium under the electrostatic fields which are necessary to keep together the ions subject to differential radiation pressure. In a star of temperature above 25,000° the equilibrium is broken down, and is replaced by a tendency for He, Ca, Si, C, N, O to be accelerated upwards in company with an appropriate number of electrons, through an upper atmosphere in which H (and possibly other elements of which we have not the atomic data for calculation) alone remains in equilibrium.

7. *Limitations to the Radiation-Pressure in an upper Chromosphere.*— We have not found any reason why an electron anywhere near a hot star should not be still subject to the radiation-pressure involved in scattering, and only a failure of the radiation-pressure on sufficient

positive ions to neutralise the loss of charge will check the ejection of electrons from an O-star. In the case of the positive ions, however, the radiation-pressure depends on conditions which outside a chromosphere may break down. We have to inquire whether a serious change in the availability of energy for absorption may occur, or whether the time during which absorption can take place can alter and make the very roughly approximate character of the thermodynamic equilibrium equations involve too serious an error when a far outer chromosphere is reached by the moving atoms.

(*a*) Milne has shown that when a large number of calcium atoms have built up a chromosphere of a certain optical thickness, the screening of this by further reinforcements from the reversing layer, and the back radiation from the chromosphere itself, will tend to maintain the whole structure constant in mass. The stability of the mechanical equilibrium will in that case depend on the growth of an absorption line just enough to control the remaining fraction (r) of photospheric radiation to that strength required to support the upper chromosphere. His first paper * gives the very great strength of absorption line needed to cut down this radiation, which would otherwise drive out Ca^- from a very hot star, a strength not seen in the lines from which we have calculated the radiation-pressure on other elements. To make $R/g<1$ we can find what value of r is needed in Table IV. For He at 30,000° the ratio would have to be reduced by a factor of the order 10^{-2}, *i.e.* the residual intensity in a line would have to be 10^{-2} of the intensity of the surrounding continuous spectrum. This would mean that the absorption lines of He in the O-star would have to be ten times as strong as the H and K lines in the sun, in order to prevent He from being expelled from the star by radiation-pressure. The faintness of absorption lines in the O-stars and their commoner tendency to appear bright, *i.e.* with a value of r greater than unity, strengthens the conviction that the solar type of chromospheric equilibrium is not reached in the hottest stars.

(*b*) The principal consequence of the breakdown of Maxwellian distribution of velocities at the top of a chromosphere will be the paucity of encounters at such a rarefaction : this will allow a longer time to elapse before each ionic recombination. The supply of electrons available for recombination will not fail, for each ionisation will liberate another, and also the electrostatic field will ensure the presence of negative charges to accompany the positive ions ; but the time between collisions (which, unlike the time for return after excitation, is not an atomic constant) may become very long.

Ionisation will be potentially as frequent as in the lower layers, since the ionising agency, here almost entirely radiative, will not fail, but the lack of recombination will mean actually a lack of ionisation to the n^{th} stage, since the frequency of the one cannot exceed the frequency of the other. The long waits will, however, allow a longer time for the chance of a $(n+1)^{th}$ stage ionisation in those atoms which have more electrons available in outer orbits. Hence there will be, at the top of the chromosphere of a very hot star, a differentiation

* *M.N.*, **84**, 354, 1924.

between He and heavier elements just as there was at a lower temperature between H and all other elements ; the impossibility of absorption for He^{++} until it recaptures an electron will mean a decrease of radiation-pressure on He in a region where encounters are rare. With heavier atoms the long times before recombination will not merely decrease the radiation-pressure through the wait before each n-fold ionisation, but will increase it through the increased possibility of $(n+1)$-fold ionisation. Thus the only limit to continued radiation-pressure on elements heavier than H and He is the availability of energy : this alone seems capable of preventing their total escape from the vicinity of the star, and perhaps allows the outflow to remain oscillating in coronal or nebular form.

8. *The Place of Hydrogen in the Atmosphere of a very Hot Star.*—In gravitational equilibrium the mass separation of elements in an atmosphere would result in H rising to the highest levels, He to the next highest, etc., and the heavier elements remaining at the base. Now, Table IV. shows that the corresponding order of possible heights in a chromosphere consisting of several elements supported by radiation-pressure will not be unique, the ratio of radiative to gravitational acceleration being capable of several turning-points in the temperature sequence, due to the combination of more than one separate factor contributing to availability of energy or to concentration in absorbing states. In the particular case of the 30,000° star, however, the order of radiation-pressure alone would leave H at the bottom of the group of elements considered ; passing through this stationary atmosphere of H there would be an outflow of He to a higher zone, while passing through both H and He zones would be an outflow of several heavier elements.

That other forces besides radiation-pressure are at work is shown by the appearance of H in the envelopes of planetary nebulæ. An important consideration here is the electrostatic theory of Rosseland.* This may possibly account for the appearance of H in regions from which its radiative properties might be expected to exclude it. Rosseland shows that under certain conditions, probably fulfilled in most stars, H will be repelled electrostatically from the core of the star, and strongly concentrated at its surface.

If the theory of Rosseland accounts for the existence of outer atmospheres of H in stars, the difference between H and other elements expected from the present calculation will appear at higher levels outside the photosphere, instead of appearing only at the base of the atmosphere where radiation-pressure alone would tend to leave the gas with only one valency electron. This difference will, however, consist as before in the tendency of H to remain oscillating about the level electrostatically decided upon, while through this stable zone will pour the heavier elements under radiation-pressure until frequencies for the next stage of ionisation become too high ; each will carry its electrical equivalent of negative ions. From the same electrical neutrality an equal number of electrons will be held back among the H nuclei, though subject to the same radiation-pressure as those which stream out of the star.

* *M.N.*, **85**, 541, 1925.

II. Velocities in the Electron capture Spectrum.

9. *Introduction.*—In Paper A it was suggested that the continuous spectrum which extends beyond the Balmer limit in certain extremely hot stars and nebulæ might be explained by the relative motions of electrons and H nuclei due to selective radiation-pressure in a gravitational field. In Part I. of the present paper the motion of ions has been treated in the light of further considerations, and in the case of various stars in the temperature sequence. We have found that the combination of selective radiation-pressure and electrostatic forces due to the high ionisation results in certain stars in a streaming out of a group of elements accompanied by an equivalent charge carried by electrons. From this streaming out H was the conspicuous exception. Part II. is an attempt to compare the relative motion of H-atoms and electrons established in Part I. with relative velocities of electrons and atoms due to other agencies, with a view to finding which sources of motion are most likely to be the origin of the continuous spectrum. For the discussion of the excitation of the spectrum through electron capture, reference must be made to Paper A.

10. *Data as to Occurrence and Extent of the Spectrum.*—The observed occurrences of the continuous spectrum beyond the Balmer limit are as follows: as absorption in A stars, as emission in the solar chromosphere and prominences, in planetary and diffuse nebulæ, and novæ.

A striking demonstration of the absorption case is the comparison spectrograms of α Lyræ and β Aquilæ by Wright.* Although the latter is a cooler star than the former, with, therefore, less intensity in the violet of the Planck temperature spectrum, its continuous spectrum nevertheless shows up far beyond the Balmer limit. This indicates that the continuous absorption in α Lyræ is a definite spectrum extending far enough for no emergence of photospheric light to be possible even as far as the instrumental possibilities allow. The emission spectrum, on the other hand, does not seem to extend so far, even in the case of objects at a far higher temperature than the A-stars; Wright mentions λ 3340 as a wave-length to which it can be followed in some planetary nebulæ.

It is worth while stating what is known and conjectured as to whether the emission occurs in the envelope or the nucleus of a planetary nebula. On p. 252 of the volume cited, Wright contrasts this spectrum in the envelope with the still further extension of a continuous spectrum in the nucleus, which, if taken as a Planck spectrum, would imply a temperature of 50,000°. On p. 215 he notices the capture spectrum in the nucleus of N.G.C. 6572, described as "As good an object as any in which to study that ultra-violet spectrum." On p. 212 Wolf is quoted as giving two maxima for the continuous spectrum of N.G.C. 6543, which would mean that the capture spectrum occurred in the nucleus, though Wright suggests that Wolf's apparatus may not be efficient in separating nucleus and envelope spectra. Stratton† considers that the spectrum is probably nuclear. This lack of certainty is perhaps not

* *Lick. Obs. Pub.*, **13**, pl. xliv.
† *Astronomical Physics*, p. 141.

of immediate importance for the physical problem of determining the conditions under which the spectrum can be excited, since even apart from the above examples the appearances in diffuse nebulæ and in novæ provide cases of both truly nebular and stellar occurrence. It is clear that whether due to capture or of Planck temperature type, the nuclear continuous spectrum of a planetary nebula extends beyond the capture spectrum of its envelope; the envelope spectrum is definite in all the photographs as having its maximum at its red end, and traceable to about λ 3340, while the nuclear spectrum generally extends to the limit of silver mirror reflection at λ 3270. This double appearance in the same star eliminates the question of atmospheric, instrumental, or photographic limitation as the reason for the termination of the envelope capture spectrum.

11. *Sources of Electron Velocities at the Edge of a Star.*—We shall consider the orders of electron velocities expected in stars, and compare them with the velocities attainable under the radiation-pressure accelerations, and with the velocities required at capture to produce λ 3340, the wave-length mentioned in the preceding paragraph.

If the electron is truly captured, the frequency ν emitted by the atom is related to the kinetic energy of the electron before capture, $\frac{1}{2}mv^2$, by the equation,

$$\nu - \nu_0 + \tfrac{1}{2}mv^2/h,$$

where h is Planck's constant and ν_0 is the frequency at the series limit, here the Balmer limit. Hence the velocity of the electron to result in emission at the Balmer limit is vanishingly small, but for emission at λ 3340 it is 3×10^7 cm. per sec.

If the electron is not captured, but switched on to a track of less energy (hyperbolic—hyperbolic transition, or "partial capture"), the frequency emitted is

$$\nu = \tfrac{1}{2}m(v_1^2 - v_2^2)/h,$$

when v_1 is its velocity before the encounter and v_2 its velocity afterwards. Hence the initial velocity of the electron to produce emission of λ 3340 by partial capture must be greater than $1{\cdot}1\times10^8$ cm. per sec., since this amount must be given up in the encounter. Whereas in total capture the amount decreases to zero as the series limit is approached from the high-frequency side, in partial capture almost the same velocity is needed to produce emission at the series limit.

(i) Consider the average velocities of electrons arising in virtue of Maxwellian distributions of thermal agitation, and the radiation which would be emitted if an electron having the velocity v, corresponding to the average energy at a certain temperature, were captured into a Balmer orbit. We have

$$\tfrac{1}{2}mv^2 = \alpha T,$$

when T is the temperature and α is the Boltzmann constant equal to $2{\cdot}06\times10^{-16}$. Let λ be the wave-length emitted on capture. The following table gives some calculated mean electron velocities:—

TABLE VI.

T°.	αT.	v in cm. per sec.	λ in A.
2,500	$\cdot 51 \times 10^{-12}$	$3 \cdot 4 \times 10^{7}$	3330
6,000	$1 \cdot 24 \times 10^{-12}$	$5 \cdot 2 \times 10^{7}$	2960
10,000	$2 \cdot 06 \times 10^{-12}$	$6 \cdot 7 \times 10^{7}$	2630
15,000	$3 \cdot 09 \times 10^{-12}$	$8 \cdot 2 \times 10^{7}$	2310
30,000	$6 \cdot 18 \times 10^{-12}$	$11 \cdot 6 \times 10^{7}$	1680

Wright has already pointed out * that very low thermal velocities could produce the observed extent of the capture spectrum in nebulæ.

(ii) Consider the velocities of photo-electric ejection of an electron by the absorption of radiation of a frequency higher than is needed for ionisation. If photospheric radiation is the agency ejecting electrons from Balmer orbits, take the wave-lengths of which the greatest intensity is available, *i.e.* the wave-length at the maximum of the black-body spectrum at certain stellar temperatures, calculated from Wien's law.

$$\lambda_{(\max.)}\mathrm{T} = \text{constant} = \cdot 29 \ (\lambda \text{ in cm.}).$$

Table VII. gives the velocities calculated in this way.

TABLE VII.

T°.	$\lambda_{(\max.)}$ in cm.	Vel. of ejection, in cm. per sec.
6,000	$4 \cdot 8 \times 10^{-5}$	No ionisation.
10,000	$2 \cdot 9 \times 10^{-5}$	$\cdot 53 \times 10^{8}$
15,000	$1 \cdot 9 \times 10^{-5}$	$1 \cdot 01 \times 10^{8}$
30,000	$0 \cdot 97 \times 10^{-5}$	$1 \cdot 8 \times 10^{8}$

(iii) If one gas is moving as a whole relative to another, encounters between the atoms of one gas and free electrons moving with the other gas may result in captures at velocities of the order of the relative mass motion. Chapman † discussed some of the possibilities of mass motions, and showed that observed drifts are probably set up by pressure diffusion, and, far less extensively, by thermal diffusion, counteracted in turn by reversions due to convection. St. John and Adams ‡ have measured velocities of gases in stars, assumed to be due to these convection currents, and found downward motions in certain stars of known temperature. These and certain other measured velocities of up-and-down drift in stellar atmospheres are tabulated for comparison with the other sources of relative motion of ions.

(iv) In the 30,000° star there is also another source of velocity peculiar to the extreme temperatures alone. If hydrogen moves in a state of comparatively small oscillations, either low in the outer surroundings of the star on the purely radiative theory, or high on

* *Nature*, **109**, 810, 1922.
† *M.N.*, **77**, 539, 1917.
‡ *Ap. J.*, **60**, 43, 1924.

Table VIII.

T°.	Star.	Velocity of Gas.
3900	Giant	$0{\cdot}34\times10^5$ cm. per sec.
5800	Dwarf	$0{\cdot}20\times10^5$,,
6400	..	$0{\cdot}67\times10^5$,,
8800	..	$1{\cdot}20\times10^5$,,
	Cepheid at maximum	$2{\cdot}4\times10^5$,,
	Cepheid at minimum	$0{\cdot}9\times10^5$,,
	Me	5 to 20×10^5 ,,
	K term in B star	4×10^5 ,,
	Nova	800 to 1800×10^5 ,,

Rosseland's theory, it remains on the whole stationary with respect to the star, with an equal number of electrons kept in the same zone by charge separation effects. Heavier elements with their electrons will stream past with accelerations of the order calculated in Part I. The H-ions will always be capturing electrons. Sometimes an ion captures one from its own zone, sometimes from the stream passing through the zone. In the latter case there is relative motion quite peculiar to the 30,000° star and to the extreme rarefaction of this outer atmosphere or "corona" of radiatively expelled gases. With accelerations of the order 2×10^5 cm. per sec.^{2}, and the extremely long free paths in this region, the relative velocities may be up to or even exceeding 10^9 cm. per sec. when an electron moving with the stream encounters an H-ion in the stationary zone.

12. *The Source of Velocity in the Continuous Spectrum.*—The velocities of mass motion seen in most stars are probably due in the main to other than radiative forces, since the large accelerations under radiation-pressure were found to be normal in stars above 25,000° only. These mass motions are probably only slightly affected by the absorption of radiation, and then temporarily and locally, as in the case of solar prominences. The velocities in Table VIII., being of the order 10^4 to 10^6 cm. per sec., are unable to produce the extensive capture spectrum. In the case of the mass motions of novæ alone we are probably dealing with radiative accelerations, since the velocities are of a higher order; the spectra here include indications of the highest temperatures.

Consider next the possibility of accounting for the observed capture spectrum by the Maxwellian thermal velocities and the photo-electric ejection velocities. In the one case when the spectrum is seen in a region where a Maxwellian distribution of velocities is assured, the reversing layer of an A-star, it appears as an absorption spectrum. Since thermodynamic equilibrium holds approximately in a normal reversing layer, the converse process of an emission spectrum at capture can be assumed to be taking place. In this case the mechanism of emission may well be the velocity of photo-electric ejection, or, alternatively, need not be sought further than the Maxwellian thermal velocity at that temperature. Tables VI. and VII. show the adequacy of either of these velocities to account for a thermodynamic equilibrium

counterpart of the absorption spectrum of α Lyræ. If velocities of thermal agitation are accepted as one of two possible sources of a capture spectrum in A-stars, they are the more probable sources in the case of the continuous emission in the solar chromosphere. Here photo-electric ejections are only derivable from a weaker part of the photospheric spectrum, and thermal agitation has not been lost in the much more extreme rarefaction of the outer atmosphere of an O-star.

In the remaining case of the continuous spectrum the physical conditions are quite different. In the truly nebular occurrences velocities will be determined by other than Maxwellian distribution. It is even probable that for any star of radiative properties corresponding to 30,000°, Maxwellian velocities cannot be relied on much above the photosphere.

Here the remaining sources of the velocities at electron capture both seem at first sight too large. The photo-electric ejections at the wave-length of maximum intensity outside a nebular nucleus are of the order of $1{\cdot}8\times10^8$ cm. per sec., and the velocity with which an electron moving with the outstreaming gases meets an ion in the stationary zone of hydrogen may very often be above 10^9 cm. per sec. On the other hand, the observed continuous spectrum due to capture outside a planetary nucleus is strongest nearest to the Balmer limit, and fades out by λ 3340. The velocities required to account for this are of the order of 10^7, unless we definitely take the step of assuming that the further nuclear continuous spectrum is due to capture: this was shown in section 10 to be so far unjustified on purely observational grounds, though in view of that possibility the temperature of the typical nucleus has in this paper been taken as 30,000°, instead of 40,000° as in Paper A.

It seems likely that the discrepancy between the huge velocities available in the combined circumstances of large free paths and the radiative accelerations discussed in this paper, and the much smaller velocities in the observed spectrum in planetary nebulæ, is an example of the inverse square law of capture probability shown by Milne * to extend from optical to X-ray frequencies. The distribution of energy along the capture spectrum is not simply due to the distribution of available velocities, but also to the fact that the probability of an electron being captured varies inversely with the square of its velocity relative to the atom. The question then arises as to what happens to the enormous velocities available, and whether they prevent capture occurring at all at the rare encounters. The answer is probably to be found in the hyperbolic—hyperbolic encounters suggested in Paper A. It is worth while inquiring how far we are likely to be able to distinguish between the importance of partial captures and true captures in an atmosphere of the planetary nebula type.

13. *Capture and Partial Capture Spectra.* —(i) Observationally the following conclusion can be stated: the spectrum which is seen is shorter than that in the absorption case in an A-star, though available velocities in the case of the planetary nebula are much greater than those which would cause emission of the wave-lengths seen in the absorption case. Hence it is clear that the planetary nebular spectrum is not a thermo-

* *Phil. Mag.*, **47**, 209, 1924.

dynamic complement to any spectrum seen in absorption. The difference between the two (apart from temperature) is mainly the breakdown of the Maxwellian velocity-distribution at very low pressures; hence it is probable that at the higher pressure of the A-star reversing layer there are unseen emissions balancing exactly each frequency in the absorption spectrum, and that in these emissions true capture is the dominant type of encounter. In the case of the planetary nebula, on the contrary, the shorter extension of the observed spectrum amid such high-velocity electrons means that partial captures are the dominant type of encounter. These encounters may be such as each to involve exchange of large proportions of the energy associated with 10^9 cm. per sec., and so to concentrate the partial capture spectrum to the violet of the Balmer limit, and perhaps contribute to the observed spectrum. The same reduction in electron speeds could be effected by a greater number of partial captures with smaller loss of energy. In the latter case the emissions would be distributed all over the spectrum and would not appear visibly in any one region.

(ii) Theoretically we can go a little further in discriminating between capture and partial capture spectra. Our experience of a partial capture spectrum in the laboratory is the emission of a continuous X-ray spectrum from material struck by a beam of cathode rays. In practice the spectrum is produced by the penetration of a bundle of rays to different distances into the material of the target, and hence observed spectra do not correspond to any unique velocity of electrons. But Kramers * has predicted the intensity distribution for a homogeneous stream of electrons, and part of his theory agrees with the distribution in a thin target deduced by Webster and Hennings † from observations with a target of finite thickness. The agreement between the theoretical isochromate of Kramers and that of Webster and Hennings shows that as the speed of cathode rays is increased past that required on the quantum law for the emission of a certain frequency by partial capture, the frequency leaps to a maximum at once and then decreases as the speed becomes higher and the energy is given to higher frequencies. This, and the shift towards higher frequencies of the maximum intensity in a continuous X-ray spectrum, due to heterogeneous rays as the voltage across the tube is increased, indicates a tendency on the part of each electron in partial capture to give up the greater part of its energy, and not to pass on with only slight diminution of velocity.

Hence it is probable that large exchanges of energy take place by partial capture, and that the extremely high velocities in the O-star are reduced before capture by high-frequency emissions, and not merely by a great many low-frequency emissions. Consider an extreme case: (*a*) If nearly all the electron motions in the nebular region were either photo-electric emissions by very high-frequency absorption, or were due to radiative acceleration over very long free paths, it is then probable that the emission due to partial capture would be concentrated in the ultra-violet. The extreme rarefaction at the boundary

* *Phil. Mag.*, **46**, 836, 1923.
† *Phys. Rev.*, **21**, 312, 1923.

of the atmosphere of a massive and extremely hot star would tend to make this type of motion predominate, and it may well be that the ultra-violet extension of the planetary nebula nuclear spectrum is thus due to partial capture. This is especially probable, since, if alternatively interpreted as a Planck spectrum it would imply a temperature far above that deduced from the ionisation theory, say, 30,000°. (*b*) Actually, however, in most stars, some other types of velocity will persist even at the boundary of the star, and some encounters will take place after free paths which are not long enough to allow of partial captures, resulting in ultra-violet emission. In O-stars which do not possess the most extreme properties the partial capture spectrum will not become noticeable at any one point, but will be spread over the whole spectrum.

Both in the extreme type (*a*) and the less extreme type of atmosphere (*b*) the energy left to the fast electrons after partial capture will be smaller than the velocities expected at that temperature in photo-electric ejection or in radiative acceleration, and it is these smaller velocities which render the electron, by the inverse square law, most liable to true capture. It is these true captures which constitute the observed spectrum immediately beyond the Balmer limit. Its extension so short a distance, and with a falling intensity in a star where such great velocities are expected, is thus due to the capture probability law and to the type of exchange of energy known to occur in partial capture. The emissions of energy in partial capture will in general be unnoticed because the range of available velocity spreads them out, but in the case of extremest rarefaction and preponderance of highest velocities they may be intense in the ultra-violet: these will form a more reasonable explanation of the nuclear ultra-violet spectrum of a planetary nebula than an explanation in terms of temperature spectra.

14. *Note on the Relation of the Foregoing to Emission Spectral Theory in General.*—Emission spectra in astrophysics are of two types: firstly, emission spectra with no background of radiation behind them, such as the flash spectrum and the spectrum of a diffuse nebula. In these cases no explanation of the appearance of emission is needed. Secondly, in bright line stars, including Me-stars, novæ, O-stars, and the nuclei of planetary nebulæ, there are emission spectra with, behind them, a non-transparent and non-reflecting emitter of approximately black-body properties, and the temperature gradient between this and the emitting atmosphere would be expected to produce absorption lines, as in the usual stellar reversing layer. Schuster's explanation of this emission as due to an extensive scattering chromosphere was examined in *M.N.*, **85**, 56, 1924, where it was found that scattering coefficients are always less than absorption coefficients, but that the pressure gradients in the stars seen to emit bright lines extend enormously the zones in which scattering has its maximum effect relative to absorption.

Now the continuous emission spectrum which we have been discussing is associated with line emissions, both in the case of no background and in the case where an explanation of the appearance in front of a background is required. But the theory of the continuous emission

we have been suggesting is not of the type required for the explanation of the line emission, *i.e.* is not of the Schuster type. The range of frequencies in the capture spectrum is due to the particular mechanism, which, as we have shown, is capable of reducing the extremely high electron velocities; these velocities are themselves peculiar to the extensive corona built up by radiation pressure. The emission does not correspond to any absorption spectrum of the same frequencies, which might be expected to appear in virtue of the temperature gradient of a more normal star, and its occurrence does not require the complex gradients which have to be called upon to explain the appearance in emission of a line which would be expected to show absorption.

Summary.

The first part of the paper is concerned with the tendencies of electrons and positive ions in a stellar atmosphere to separate under differential gravitational forces and under differential radiation-pressure, and the equilibrium reached between these tendencies and the electrostatic fields set up by separation of charges. The breakdown of this equilibrium, under certain maximum conditions of selective radiation-pressure, is calculated by an extension of the method used in a previous paper on hydrogen and helium; it is found that such breakdown only becomes probable when a star has an effective temperature above 25,000°, and that it affects the elements He, Ca, Si, C, N, O. The electrostatic forces between ions allow a stream of atoms of these elements to move at high speed through the outer chromosphere of such a star, together with an equal charge carried by electrons. Hydrogen, however, remains comparatively stationary at the boundary of the star.

In the second part, the electron captures made by the hydrogen are considered. The velocity at capture implied by the continuous spectrum of the envelope of a planetary nebula is compared with the available velocities. These are calculated from thermal agitation, photo-electric ejection, and convection currents in stars. The encounters at very high velocity between hydrogen atoms and the electrons which accompany the elements expelled by radiation-pressure are primarily partial captures. The effect of partial captures is related through the X-ray continuous spectrum to the continuous spectra of both envelope and nucleus of a planetary nebula. It is pointed out that this theory of continuous emission is different from the theory required for line emission spectra.

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The Reflection Effect in Eclipsing Variables.

By A. S. Eddington, F.R.S.

1. In some of the eclipsing variables the components are so close that reflection by the faint star of the light of the bright star is conspicuous in the light-curve. The hemispheres turned towards and away from the bright component are of unequal luminosity. For this reason, as well as on account of the spheroidal distortion of the two stars, the light is not constant between eclipses. The reflection effect and the spheroidal effect are easily disentangled by their difference of phase, and it is the practice to determine and allow for both when the dimensions of the system are investigated by the method of Russell and Shapley. We shall here consider the physical theory of the reflection effect.

For the moment we pass over the difficulty of translating a light-curve into a heat-curve and confine attention to the amount of energy reflected. The theory is then very simple. *A star necessarily re-emits completely the radiation falling on it.* In other words, the heat-albedo $=1$.

This is obvious when we realise that the heat ordinarily emitted by the star is determined by the temperature gradient in the deep interior. The peripheral regions containing little mass cannot appreciably dam or reinforce this outflow; they can only adjust their temperature and density so as to let it pass. Consider a sphere S enclosing, say, 99 per cent. of the mass of the star; the outflow L which crosses S is fixed by conditions interior to S. There will be the same net outflow across any larger sphere S′, since there is no appreciable source or sink of energy between S and S′. If, then, radiation L_i from another star passes inwards across S′, the gross outflow across S′ is increased to $L+L_i$ so as to maintain the same net outflow L. This conclusion would fail if the incident radiation affected the temperature and density interior to S, and so altered L. Theoretically the star has to change its equilibrium slightly so as to accommodate itself to the increased value of radiation-pressure at the boundary. But the increase is at most 10 dynes per sq. cm., and as the pressures within S are above 10^8 dynes per sq. cm. the effect on the interior is found to be quite negligible.

The above argument amounts to this—we shall not appreciably alter the internal state of a star by lightly patting its surface, so that the stream of energy continues to pour out from it at the normal rate; and since it is the net outward flow which is thus prescribed, the gross outward flow is greater by the amount of the incident radiation. It is convenient to speak of this additional radiation as "reflected," but the actual process is absorption followed by re-emission (or possibly scattering).

2. We now calculate the effect on the brightness at full phase. Let L_1, L_2 be the luminosities of the two components S_1 and S_2, and let r_1, r_2 be their radii. Let a be the distance between their

centres P_1, P_2. Consider an annulus QQ′ on S_2, and let $QP_1P_2=\phi$, $QP_2P_1=\theta$ (fig. 1).

The amount of radiation from S_1 falling on the annulus is

$$\tfrac{1}{2}L_1 \sin\phi d\phi \quad . \quad . \quad . \quad . \quad (1)$$

and this must be re-emitted by the annulus in addition to its ordinary radiation, which amounts to

$$\tfrac{1}{2}L_2 \sin\theta d\theta \quad . \quad . \quad . \quad . \quad (2)$$

To find the brightness as seen from a great distance in the direction

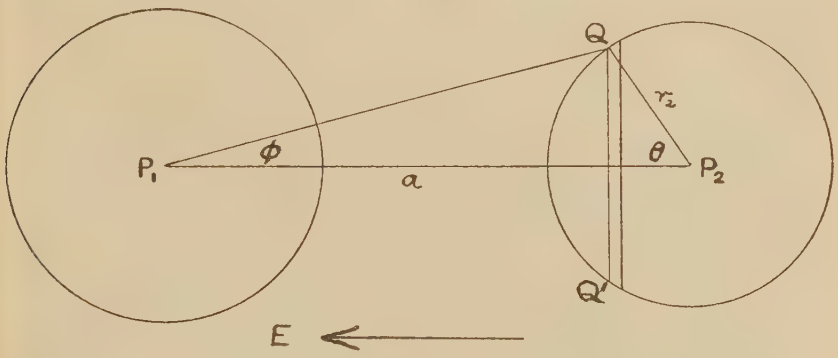


Fig. 1.

E we must multiply by the foreshortening factor $\cos\theta$ and integrate. The reflected radiation is to the ordinary radiation in the ratio

$$\tfrac{1}{2}L_1\int_0^{\phi_0} \sin\phi\cos\theta d\phi \quad : \quad \tfrac{1}{2}L_2\int_0^{\frac{\pi}{2}} \sin\theta\cos\theta d\theta \quad . \quad . \quad (3)$$

where $\sin\phi_0=r_2/a$. It can be shown that

$$\cos\theta=\{\sin^2\phi+\cos\phi\sqrt{(\cos^2\phi-\cos^2\phi_0)}\}/\sin\phi_0,$$

and the ratio (3) becomes

$$\frac{2}{3}\frac{L_1}{L_2}\{\sin^2\phi_0+(2+\cos^3\phi_0-3\cos\phi_0)/\sin\phi_0\} \quad . \quad . \quad (4)$$

When ϕ_0 is not too large this is approximately $\frac{2}{3}(L_1/L_2)(r_2/a)^2$, but it is generally desirable to use the more accurate formula.

3. The actual ratio is a little larger than (4) owing to "darkening at the limb," which affects the ordinary radiation more than the radiation reflected at full phase. It will be sufficient to calculate this minor correction when r_2/a is small, so that the reflected and ordinary radiation at Q are in the ratio

$$L_1\cos\theta/a^2 \quad : \quad L_2/r_2^2.$$

Then we multiply by the darkening factor $\{1-\alpha(1-\cos\theta)\}$ as well as

by the foreshortening factor $\cos\theta$ and integrate over the hemisphere of the star. The ratio is

$$\frac{L_1}{a^2}\int_0^{\frac{\pi}{2}}\cos^2\theta(1-\alpha-\alpha\cos\theta)\sin\theta d\theta \;:\; \frac{L_2}{r_2^2}\int_0^{\frac{\pi}{2}}\cos\theta(1-\alpha-\alpha\cos\theta)\sin\theta d\theta$$
$$= \frac{2}{3}\frac{L_1}{L_2}\frac{r_2^2}{a^2}\frac{1-\frac{1}{4}\alpha}{1-\frac{1}{3}\alpha} \qquad (5)$$

The theoretical value of α (verified by measurements on the solar disc) is 0·6. The correcting factor $(1-\frac{1}{4}\alpha)/(1-\frac{1}{3}\alpha)$ is 17/16. This enhanced luminosity for full phase is at the expense of the luminosity in other directions, since the mean albedo is unity as explained in § 1.

We have assumed here that the darkening factor is the same for the reflected as for the ordinary light. The conditions are, however, considerably different, and I have consulted Professor Milne as to the effect of these. He furnished me with an unexpectedly simple solution of the problem (which I hope he will publish). I find however that, using his formulæ, the correcting factor is still 17/16.

4. Still considering r_2/a to be small, and now neglecting limb-darkening, we examine how the brightness will change with phase. First consider half phase so that the direction P_2E (E being the earth) is at right angles to P_2P_1; the foreshortening factor will be cos QP_2E instead of $\cos\theta$. Regarding $\cos\theta$ and cos QP_2E as direction cosines l, m, the brightness at full phase contains the factor l^2 integrated over a hemisphere and at half phase contains lm integrated over a quadrant. The ratio is $1/\pi$.

The general formula can be found without much trouble. If ψ is the phase angle measured from "new" ($\psi=\pi-P_1P_2E$), the brightness of the reflected radiation is proportional to

$$\sin\psi-\psi\cos\psi \quad . \quad . \quad . \quad . \quad (6)$$

5. Published values of the reflection coefficient deduced from photometric observations have been computed on the assumption that the brightness varies as $1-\cos\psi$ instead of (6). As it happens, this introduces no inaccuracy because in order to separate the reflection effect from the spheroidal effect the determination is made to depend on the difference of brightness at phases ψ and $\pi-\psi$. For such comparisons the expressions $\frac{1}{2}(1-\cos\psi)$ and $\frac{1}{\pi}(\sin\psi-\psi\cos\psi)$ are equivalent. But the ellipticity as ordinarily computed will be appreciably in error. In the usual notation the luminosity between eclipses is analysed for the terms

$$-b\cos\psi-\tfrac{1}{2}c\cos 2\psi.$$

Analysis of (6) gives the terms

$$-b\cos\psi+\frac{16}{9\pi^2}b\cos 2\psi,$$

so that the true ellipticity coefficient c' is

$$c' = c + \frac{32b}{9\pi^2} = c + 0{\cdot}36b.$$

This conclusion that the reflection effect partially masks the ellipticity effect was also reached by J. Stebbins (*Seeliger Festschrift*, p. 442).

Collecting together the results of §§ 2, 3, 4, we see that if L_1, L_2 are the ordinary luminosities of the primary and secondary the additional luminosity of the latter due to reflection is

$$L_r = \frac{2}{3} \cdot \frac{17}{16} \cdot L_1 \{\sin^2\phi_0 + (2 + \cos^3\phi_0 - 3\cos\phi_0)/\sin\phi_0\}(\sin\psi - \psi\cos\psi)/\pi \quad (7)$$

subject to inaccuracies due to treating the various developments separately instead of simultaneously.

6. Up to the present our measurements are supposed to be in heat units, and the "luminosity" is really the heat intensity. Unfortunately the practical measurement of the reflection coefficient refers generally to light intensity (occasionally to photo-electric intensity) and the theory gives less definite assistance. We can, however, decide that for most of the stars *the reflection coefficient for light will not be less than the reflection coefficient for heat.*

The simplest assumption is that the incident radiation after being absorbed is re-emitted along with the ordinary radiation, the whole constituting black-body radiation for increased temperature. Usually the effective temperature of S_1 can be inferred from its observed spectral type, and that of S_2 from the ratio of surface brightnesses of the two stars found from analysis of the light-curve. The consequences of this assumption can thus be worked out fully for each system.

Generally in variables showing the reflection effect the star S_1 is of type B8–A, with effective temperature about 12,000°. The surface brightness of S_2 is about $\frac{1}{10}$ that of S_1, so that its effective temperature is about 6000°. The temperature of the bright hemisphere of S_2 will be increased to 7000° or 8000° by the radiation which it is reflecting. The temperature of maximum luminous efficiency is in the neighbourhood of 7000°, so that when the incident radiation (originating) at 12,000° is re-emitted at about 7000° its luminous efficiency will be increased—actually about 70 per cent. The proper radiation of S_2 will also have its luminous efficiency slightly altered by the increase of temperature of emission; in the case here considered the change is small; sometimes when the original temperature is lower there is a considerable increase, and it rarely happens that the original temperature is so high as to allow a decrease.

The result is that in general there is an increase of luminous efficiency, so that the reflection effect in light units is greater than in heat units.

7. I have collected in the table the results for those variables in which the reflection effect has been closely studied. No. 8 is by J. Stebbins (*Ap. J.*, **32**, 185); it is based on selenium photometry—a circumstance to be remembered if it is desired to transform from

photometric to heat measurement. The others, based on visual observations, are taken from *Princeton Publications*, Nos. 1–7;* stars 3 and 7 are due to R. J. McDiarmid and the rest to R. S. Dugan.

Reflection Effect.

No.	Star.	Type.	J_1/J_2.	L_1.	r_1/a.	r_2/a.	L_r (calc.).	L_r (obs.).	C–O.	P.E.
1	SZ Her.	..	4·3	·800	·318	·331	·078	·030	+·048	
2	U Cep.	A	13·5	·838	·191	·308	·069	·043	+·026	·009
3	TV Cas.	B9	6·9	·849	·275	·302	·068	·074	−·006	·008
4	RZ Cas.	A	12·2	·902	·253	·288	·064	·063	+·001	·008
5	RT Per.	..	4·8	·863	·309	·269	·053	·022	+·031	·009
6	Z Dra.	..	12·8	·911	·238	·263	·053	·040	+·013	·006
7	TX Cas.	B	1·3	·829	·519	·270	·052	(·030)	(+·022)	
8	β Per.	B8	11·6	·895	·210	·239	·043	·045	−·002	
9	RS Vul.	B8	7·0	·804	·201	·262	·048	·078	−·030	
10	R CMa.	F	13·2	·934	·245	·236	·046	·015	+·031	
11	Y Cam.	A–F	20·0	·955	·236	·225	·041	·041	·000	
12	RV Oph.	A	12·2	·825	·125	·200	·027	·018	+·009	·006

The columns J_1/J_2 (ratio of surface brightness) and r_1/a are given in order to form an idea of the nature of the system. The normal light of S_2 is $1-L_1-L_r$ (obs.). The reflected light at full phase L_r, or difference between the bright and dull hemispheres of S_2, is calculated from L_1 and r_2/a by (4), the correcting factor $\frac{1}{16}$ being applied. This is compared with the observed value of L_r as given by the authorities responsible for the orbit. The probable error of the observed value if known is given in the last column. When "uniform" and "darkened" solutions are given, these have been weighted as 2 : 3 to correspond with $\alpha=0{\cdot}6$. When the elongation is sensible, the minor axis has been used for r_2.

8. We really need an individual scrutiny of each star, since the light-curves are by no means equally favourable for determining the reflection effect. However, we confine ourselves to a general survey. Omitting No. 7, the agreement is good for 6 stars, 4 stars give large positive residuals, and RS Vul. is exceptional in showing a large negative residual.

The column L_r (obs.) should, I think, be multiplied by $(1+J_2/J_1)$ to allow for reflection of the light of S_2 by S_1, which partly cancels the other reflection. This chiefly affects No. 7, which would be thereby brought into agreement.

The most trustworthy of the discordant stars are U Cep. and RT Per. The former would perhaps admit a higher value of L_r without straining the observations, but the latter seems definitely outstanding. The observed value of L_r for RS Vul. seems to be very well determined. According to § 6 we anticipate a negative residual on account of the change of luminous efficiency on re-emission, so that taken by itself we should not regard RS Vul. as discordant. But if that is the explana-

* See also revised elements in *Ap. J.*, **39**, 405.

tion, why does not the same excess appear in any of the other stars ? The weight of the evidence of the table is against admitting the change of luminous efficiency, and I suspect that the dimensions of this system (which has always been found difficult to treat) have not yet been correctly settled.

9. The table seems to have decided by a vote of 11 to 1 that the expected increase of luminous efficiency has not materialised. It looks as though the high-frequency radiation from the primary is sent back from the high-level regions of the faint star in much the same wave-lengths as are incident. Several suggestions can be made to account for this, but I do not feel prepared to follow them up in detail.

Closely connected with this is the problem of the spectral type of the secondary. The comparatively late type (G–K) corresponding to its surface luminosity does not seem to be borne out by direct spectroscopic observation. The difference of type could scarcely be overlooked by anyone measuring the radial velocity of S_2 ; and the silence of spectroscopic observers as to the difference of type is as significant as the explicit statements occasionally made that the types of S_1 and S_2 are nearly the same.

A little consideration shows that the spectrum of the hemisphere of S_2 turned towards a star S_1 with higher temperature is likely to approximate to the spectrum of S_1, even when the incident radiation is relatively weak. The radiation from S_1 will ionise and excite atoms in S_2 very much as it does in S_1 itself, and these will absorb their appropriate lines in the outflowing radiation from S_2. Two points should be noticed :

(1) The result is not merely a reflected spectrum of S_1 superposed on that of S_2 ; *all* the radiation from the bright hemisphere of S_2 is subjected to the absorption, and it may be relieved from the absorption by un-ionised atoms which would have occurred if S_2 were isolated.

(2) The radiation from S_1 has a strategic advantage in that it attacks the atmosphere of S_2 from outside. The absorption for which it is responsible is the last to be imprinted on the spectrum of S_2, and cannot be blurred out by any subsequent experiences of the radiation.

10. For two of the foregoing stars RS Vul. and TV Cas. spectrographic orbits of both components have been found, and the masses are known. It was shown in *M.N.*, **84**, 318, that the results for the bright components agree fairly well with my mass-luminosity curve.* Owing to the exceptionally thorough photometric study of these stars, a closer examination seems desirable. Unfortunately there is still a mystery about RS Vul., and it would be misleading to use it until the question is cleared up ; but TV Cas. is a well-behaved star and ought to give good results.

Combining Plaskett's orbit (*Pub. Dom. Obs. Vict.*, **2**, 141) with McDiarmid's photometric solution (*Princeton Pub.*, No. 7), we have

$$M_1=1{\cdot}74, \quad M_2=1{\cdot}02, \quad r_1/a={\cdot}279, \quad r_2/a={\cdot}306,$$
$$r_1=2{\cdot}45\times\odot, \quad r_2=2{\cdot}68\times\odot, \quad L_1/L_2=11{\cdot}2, \quad J_1/J_2=13{\cdot}6,$$

* The residual there given for TV Cas. is too small, a computational error having been made.

where L_2 and J_2 refer to the faint hemisphere of S_2. Taking the effective temperature of S_1 to be 10,500°, corresponding to type B9, the ratio J_1/J_2 gives 5200° for S_2.

Then the absolute mags. are computed (1) directly from r_1, r_2, and the temperatures; (2) by the mass-luminosity relation from M_1, M_2, and the temperatures. The results are:

TV *Cassiopeiæ*.

	Temp.	Bol. Mag. (from Radius).	Bol. Mag. (from Mass).	Residual.
S_1	10,500	+0·34	+1·86	1·52
S_2	5,200	+3·18	+4·58	1·40

The agreement is not so good as expected, but I believe that there is a simple explanation. Plaskett does not note any difference in the spectra of the two components, and in so far as the lines which he measured in S_2 are characteristic of the A-type spectrum they could only be produced in the illuminated hemisphere. In any case, one hemisphere is twice as bright as the other, so that the measures will not refer to the centre of the disc. Since $r_2 = \cdot 306a$ the displacement may well amount to $\cdot 08a$, and the radius of the spectroscopic orbit is therefore to be increased in the ratio 100 to 92. This gives $M_1 = 2 \cdot 35$, $M_2 = 1 \cdot 20$, and the comparison now runs:

	Mag. from Radius.	Mag. from Mass.	Residual.
S_1	+0·16	+0·81	0·65
S_2	+3·00	+3·91	0·91

The agreement is now sufficiently close considering the nature of the data.

11. Although not connected with reflection effects, I may add here another comparison of an eclipsing variable with the mass-luminosity curve. The star Algol has not hitherto been used for a comparison, because the spectrum of the secondary had not been observed and the absolute dimensions were therefore unknown. Linear dimensions of the stars and their orbits with the resulting masses are often quoted, and perhaps it has sometimes been overlooked that these are merely hypothetical figures given originally for purposes of illustration. But in 1924 a genuine determination of the size of the Algol system was made by a remarkable new method developed at Ann Arbor Observatory by R. A. Rossiter and D. B. McLaughlin (*Ap. J.*, **60**, 15, 22). At different stages of the main eclipse different portions of the disc of the primary are exposed, and advantage is taken of this to measure the *differential radial velocity* between the two limbs, and hence the equatorial velocity of rotation. When the components are so close as in Algol it can be assumed that they keep the same face turned towards one another, so that the period of rotation is the same as the orbital period. From the rotation period and equatorial velocity the radius of Algol is found to be

2,180,000 km.,

about double the value commonly assigned. With the aid of photometric data, other constants of the system can now be found; in particular, the mass ratio is 5·0 : 1. The result appears to be of considerable accuracy, and the method is likely to be of great importance.

I was not acquainted with this paper until a month ago, and it so happens that I had sent to press a few weeks earlier a calculation of the dimensions of Algol according to the mass-luminosity relation—not expecting that there would be any means of putting the theoretical prediction to an observational test. On referring to the proof-sheets I found that my calculated radius was 2,140,000 km. The conventional figures for Algol had, of course, been hopelessly discordant with the mass-luminosity relation.

Summary.

1. The heat-albedo of a star reflecting radiation from another star is unity. If the heat reflected in a particular direction is being considered, the albedo at full phase is $\frac{17}{18}$.

2. If c is the ellipticity coefficient as usually found from the observations, and b the reflection coefficient, the true ellipticity coefficient is $c+0{\cdot}36b$.

3. In 12 systems examined the calculated heat-reflection coefficient agrees with the observed light-reflection coefficient in 7 systems; it is greater in 4 systems, and less in RS Vulpeculæ. We had rather anticipated that owing to the difference between heat-albedo and light-albedo all stars would deviate in the same sense as RS Vulpeculæ; but since this anticipation is not confirmed by the other stars it would seem that the solitary exception must be due to erroneous observational data.

4. If the re-emitted radiation is amalgamated with the natural emission into black-body radiation, the circumstances are nearly always such as to increase the luminous efficiency. In typical cases the increase would be 70 per cent. Since the increase is not confirmed by observation, we infer that the light is re-emitted by some mechanism which preserves its original quality.

5. The luminosity of the two hemispheres of the secondary is often very unequal, so that radial-velocity measurements will not refer to the centre of the disc. This should be taken account of in determining the orbits and masses. The accordance of TV Cassiopeiæ with the mass-luminosity curve is greatly improved by taking account of this correction.

6. Reference is made to a determination of the dimensions of Algol by D. B. McLaughlin by a new method. The results agree precisely with a calculation made from the mass-luminosity curve.

On Radiative Viscosity and the Rotation of Astronomical Masses.

By J. H. Jeans.

1. The ordinary discussion of figures of equilibrium of rotating masses proceeds on the assumption that the masses are rotating like rigid bodies, so that each element moves with the same angular velocity about the axis of rotation. It has long been known that ordinary gaseous viscosity is quite inadequate to ensure this uniformity of rotation. The present paper shows that the flow of radiation, carrying momentum with it from one part of a star to another, provides a more potent agency than ordinary gaseous viscosity. This new agency, which we shall call "radiative viscosity," bears the same relation to ordinary viscosity that Schwarzschild's radiative transfer of heat bears to ordinary heat-conduction. The recognition of its existence brings new factors into such problems as stellar rotations, the fission of stars by rotation, and the dynamics of the spiral nebulæ.

2. In a gas in which all the molecules had the same velocity c and the same free path l, the coefficient of viscosity would be given by the usual simple equation *

$$\kappa = \tfrac{1}{3}\rho c l \quad . \quad . \quad . \quad . \quad . \quad (1)$$

and the coefficient of conduction of heat would be †

$$\kappa C_v \quad . \quad . \quad . \quad . \quad . \quad (2)$$

where C_v is the specific heat of the gas. Thus in a temperature-gradient $\partial T/\partial x$ the flow of heat per square centimetre would be

$$\tfrac{1}{3}\rho c l C_v \frac{\partial T}{\partial x} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

This equation is true whether the carriers are molecules, atoms, or free-electrons, except in so far as the numerical constant may require adjustment in consequence of special properties of these carriers. Under stellar conditions the main flow of heat is by radiation, so that the important carriers are rays of light or of high-frequency radiation. The specific heat per unit volume of these carriers is now $\frac{\partial}{\partial T}(aT^4)$, or $4aT^3$, in place of ρC_v which figures in formula (3), while the velocity of the carriers is C, the velocity of light, in place of c. If l is the mean free path in ordinary gas theory, a stream of particles is reduced to $e^{-x/l}$ of its original strength after travelling a distance x.‡ And if k is the coefficient of opacity of stellar material, a beam of radiation is reduced to $e^{-k\rho x}$ of its original strength after travelling a distance x. Thus the free path of the radiation must be taken to be $1/k\rho$. If we now make the appropriate substitutions in formula (3), namely, $4aT^3$

* Jeans, *Dynamical Theory of Gases*, p. 273.
† *Loc. cit.*, p. 291.
‡ *Loc. cit.*, p. 257.

for ρC_v, C for c, and $1/k\rho$ for l, we find for the flow of heat per square centimetre which is carried by radiation,

$$\frac{4}{3}\frac{aT^3C}{k\rho}\frac{\partial T}{\partial x} \quad . \quad . \quad . \quad . \quad (4)$$

Since $aC=4\sigma$ where σ is Stefan's constant, this takes the form

$$\frac{16}{3}\frac{\sigma T^3}{k\rho}\frac{\partial T}{\partial x} \quad . \quad . \quad . \quad . \quad (5)$$

which is identical with the formula for radiative transfer derived by Eddington from Schwarzschild's theory.

3. The phenomenon of viscosity can be treated in a similar way, rays of radiation now being regarded as carriers of momentum. To obtain the radiative coefficient of viscosity κ_R, we transform equation (1) by writing aT^4/C^2 for ρ, the mass of the carriers per unit volume, and, as before, C for c and $1/k\rho$ for l. In this way we obtain

$$\kappa_R=\frac{aT^4}{3Ck\rho}, \quad . \quad . \quad . \quad . \quad (6)$$

the radiative flow of momentum per sq. cm. being

$$\kappa_R\frac{\partial w}{\partial x}.$$

The ratio of the coefficients of radiative viscosity and ordinary viscosity can be put in the form

$$\frac{\kappa_R}{\kappa}=\left(\frac{aT^4}{\rho c^2}\right)\left(\frac{c}{C}\right)\left(\frac{1}{k\rho l}\right),$$

and on inserting numerical values we find that the largeness of the last factor makes κ_R/κ large in all ordinary stars. Radiation transports less mass than ordinary material transfer, but it carries this mass much faster, so that so far the amounts of momentum transferred per unit time are not hopelessly unequal. The decisive factor is the free-path, radiation having a longer free-path than material carriers of momentum. This is true as regards transfer by molecules, by atoms or nuclei, or by free-electrons as recently investigated by Persico.*

For instance, at the centre of the Sun the values of T and ρ are approximately† $T=3{\cdot}3\times10^7$ and $\rho=36$, and k is about 180.

Equation (6) now shows that the value of κ_R is about 16. This may be compared with the coefficient of ordinary gaseous viscosity, which is about 10^{-4} at ordinary temperatures, and about unity at

* *M.N.*, **86** (1926), p. 93.

† These agree with the values used by Persico, being deduced from Eddington's calculations. I have used these so that my $\varkappa_R$ shall admit of direct comparison with the value of $\varkappa$ found by Persico. Otherwise (*cf.* § 7 below) I would have preferred to assign substantially higher values to both T and ρ. These higher values would have led to a slightly higher value for $\varkappa_R$

stellar temperatures, or with the value 4·2 calculated by Persico for the viscosity produced by the motion of free-electrons.

Just as formula (1) for gaseous viscosity is subject to a numerical correction required by the special properties of the molecular carriers—unequal velocities, "persistence" of velocities, etc.—so formula (6) for radiative viscosity is subject to numerical correction arising from the circumstance that the velocities of light and material media are not compounded according to ordinary vector laws. For this reason the numerical results of the present paper are approximate only.

4. The theory of Kramers assigns to k the value

$$k=\frac{c\rho}{\mu T^{3\cdot5}}\left(1+\frac{h\nu}{RT}\right) \quad . \quad . \quad . \quad . \quad (7)$$

where μ is the effective molecular weight of the stellar material. Inserting this value of k into equation (6), we find

$$\kappa_R=\frac{a\mu T^{7\cdot5}}{3cC\rho^2}\left(1+\frac{h\nu}{RT}\right)^{-1} \quad . \quad . \quad . \quad (8)$$

At first sight κ_R appears to vary very rapidly with both T and ρ. But in a star of small mass (*cf.* § 7 below) ρ varies approximately as $T^{3\cdot25}$, so that the variations of T and ρ largely cancel one another in the interior of a single star, and κ_R varies approximately as T. In a star of very large mass ρ varies as $T^{3\cdot5}$, so that κ_R varies still less, being now approximately proportional to $T^{0\cdot5}$. In all stars κ_R is largest at the centre and falls towards the surface.

As we pass to stars of higher mass, the ratio of T^3/ρ which measures the ratio of the pressure of radiation to the total pressure increases. Thus the value of κ_R corresponding to a given value of T increases, and calculation shows that in general the value of κ_R at the centre of the star increases. At the centre of Capella we have approximately *

$$k=47\cdot5, \quad \rho=\tfrac{1}{7}, \quad T=10^7,$$

so that κ_R is found to be about 125. In a giant star κ_R is much larger than in the Sun, while κ, the coefficient of ordinary material viscosity, is less than in the Sun, its value being independent of the density but decreasing with decrease of temperature. Thus the preponderance of radiative viscosity is far more marked in giant than in dwarf stars.

5. The general equations of motion for stellar matter are

$$\rho\frac{Du}{Dt}=\rho X-\frac{\partial p}{\partial x}+\kappa\left(\frac{1}{3}\frac{\partial\theta}{\partial x}+\nabla^2u\right), \text{ etc.} \quad . \quad . \quad (9)$$

where κ denotes the coefficient of radiative viscosity, or, more accurately, the sum of this and the coefficient of material viscosity.

A well-known theorem of Poincaré † shows that a star in which p is a function of ρ only cannot rotate in a steady state unless ω the

* These are Eddington's values (*M.N.*, **84** (1924), p. 109), to which the footnote on p. 329 again applies.

† *Les Hypothèses Cosmogoniques*, p. 28 ; or Jeans, *Problems of Cosmogony*, p. 207.

angular velocity is a function only of ϖ, the perpendicular distance from the axis of rotation; the rotation must be by cylinders about the axis. If κ is treated as a constant, I have shown * that in such a case the angular velocity ω must equalise itself as directed by the equation.

$$\frac{d\omega}{dt}=\frac{\kappa}{\rho}\nabla^2\omega=\frac{\kappa}{\rho\varpi^2}\left(\varpi\frac{\partial}{\partial\varpi}\right)^2\omega \quad . \quad . \quad . \quad (10)$$

which is the well-known Fourier equation of conduction of heat.

In an actual star, p will not be accurately a function of ρ, and κ will not be accurately constant, but in a general way inequalities of angular velocity will disappear at about the rate predicted by equation (10). Thus inequalities at a radial distance ϖ apart will be reduced to half value in a time of the order of

$$\frac{\rho\varpi^2}{\kappa} \quad . \quad . \quad . \quad . \quad . \quad (11)$$

At the centre of Capella $\kappa=125$ while ρ is $\frac{1}{7}$. The radius of Capella is about 8×10^{11}. Thus the time for inequalities to reduce to half over a distance equal to a quarter of the radius of Capella is of the order of $4{\cdot}6\times10^{19}$ seconds or $1{\cdot}5\times10^{12}$ years. At the centre of the Sun $\kappa=16$, while ρ is 36; the Sun's radius is 7×10^{10}. Thus the time for inequalities to reduce to half over the quarter radius of the Sun is of the order of 7×10^{20} seconds or $2{\cdot}2\times10^{13}$ years.

The time for the Sun is longer than the probable total age of the Sun, so that there can have been but little equalisation of ω in the Sun in recent ages. For giant stars, such as Capella, the possibilities of equalisation are somewhat greater, but even here there can be nothing approaching complete equalisation within the duration of the giant stage. In a general way it seems likely that an imperfect equalisation may take place in the giant stage, and that any surviving inequalities of angular velocity will persist through the dwarf stage.

The foregoing numerical calculation had reference to the central regions of the two stars considered. The time of equalisation over a specified distance is proportional to ρ/κ, which, from equation (8), is proportional to $\rho^3T^{-7{\cdot}5}$. As ρ varies approximately as $T^{3{\cdot}25}$ inside a star, ρ/κ must vary approximately as $\rho^{0{\cdot}7}$. Thus the process of equalisation will be somewhat more rapid in the outer layers of a star than in its central regions.

6. We have so far considered the process of smoothing out inequalities of angular velocity; we now turn to a process which continually creates these inequalities. Consider a sphere of radius r drawn inside a star, and let energy be produced inside this sphere by the annihilation of matter at the rate of a mass m per unit time. Each unit time a mass m of radiation crosses the sphere, and if the sphere is rotating as a rigid body with angular velocity ω, the mass m of radiation carries with it angular momentum $\frac{2}{3}mr^2\omega$. A shell bounded by spheres of radii r and $r+dr$, consequently receives angular momentum of this

* *M.N.*, **77** (1917), p. 201.

amount per unit time, and discharges angular momentum in the form of radiation of amount

$$\tfrac{2}{3}m(r+dr)^2(\omega+d\omega).$$

Its own angular momentum is $\frac{8}{5}\pi\rho r^4 dr\omega$, so that, equating the decrease in its angular momentum to its net loss by radiation, we obtain

$$-\frac{d\omega}{dt}=\frac{1}{3}\frac{\bar{\rho}}{\rho}\frac{m}{M_r}\left(2\omega+r\frac{\partial\omega}{\partial r}\right) \quad . \quad . \quad . \quad (12)$$

where M_r is the total mass inside the sphere of radius r and $\bar{\rho}$ is its mean density. We have here regarded ω as a function of r, but as the greater part of the angular momentum of a rotating shell comes from the region farthest from the axis, r may be replaced by ϖ without introducing any great error.

This equation shows that the value of ω changes throughout the star, generally in the direction of decrease, as the result of the passage of radiation through the star. Radiation is generated inside the star by the annihilation of matter which, being situated fairly near the centre, had but little angular momentum. At first the radiation has precisely the same angular momentum as the matter it replaces, but as it passes outward through the star its angular momentum is continually increased and that of the star suffers accordingly.

If we neglect variations of angular velocity throughout the star, and consider a shell in which ρ is equal to $\frac{2}{3}\bar{\rho}$, equation (12) becomes

$$-\frac{d\omega}{dt}=\omega\frac{m}{M_r}.$$

Since m is defined to be equal to $-dM_r/dt$, this has the integral

$$\frac{\omega}{M_r}=\text{constant} \quad . \quad . \quad . \quad . \quad (13)$$

Thus in a general way the angular velocity of a star decreases at about the same rate as its mass. This conception has some bearing on certain problems, such as tidal friction, the formation of binary stars by fission, and the origin of the solar system, in which it is assumed that a rotating body retains a constant moment of momentum. For our present purpose we notice that even if ω is initially constant through the star, equation (12) shows that it cannot remain constant; in general it decreases most rapidly where ρ is least, namely, in the star's outer layers. Thus while viscosity is smoothing out inequalities of angular velocity, new inequalities are forming as a result of the flow of radiation through the star.

7. It is now clear that through the whole of its life a star can never rotate as a rigid body, a conclusion to which the present state of the Sun bears testimony. We must think of a star as a collection of concentric rotating layers, each rotating at a different speed. And just as the temperature at every point of a star is determined by the inflow and outflow of heat carried by radiation, so the angular velocity of each

element of the star is determined by the inflow and outflow of the momentum carried by radiation.

This conclusion has a far-reaching effect on the dynamical theory of the rotational fission of stars. The established theory proceeds on the tacit assumption that viscosity is so potent as to secure a uniform velocity of rotation throughout the star at every instant. We have now seen that it cannot secure this during the whole life of the star, and still less so, of course, during the comparatively short period in which fission is in progress. New factors come into the problem, while some old ones drop out. The old supposition that the star is rotating as a rigid body must be replaced by a new supposition that the layers of the star are moving like a non-viscous fluid. The effect on the theory of the Maclaurin spheroid is not very great, but on the theory of the Jacobian ellipsoid it is fundamental.

For some time it has been apparent that the only hope of reconciling rotational theory with observation lay in the introduction of some entirely new physical factor. If λ is defined by

$$(\text{gas-pressure})=\lambda\times(\text{pressure of radiation}) \quad . \quad . \quad (14)$$

at any point in the interior of a star, I have shown * that λ varies approximately as $T^{\frac{1}{2}}$ in a star of small mass. Inserting this value of λ into equation (14) and replacing the other factors by their values in terms of T and ρ, we find that ρ varies as $T^{3\cdot25}$, so that the star is arranged like a mass of gas in adiabatic equilibrium with γ, the ratio of the specific heats, equal to 1·308, the result already quoted. In a star of large mass the variation of λ with T is still more rapid, so that the value of γ is even less, reaching the value $\gamma=1\cdot286$ in the limiting case of a star of infinite mass. Such values of γ imply an enormous degree of central condensation. Emden's numerical solutions † show that even with the value $\gamma=1\cdot308$, the star's central density is well over 100 times its mean density, while at least half the total mass is contained in a sphere of radius one-fifth of that of the star and so of volume only 0·008 of that of the star. The smaller values of γ which occur in giant stars involve an even higher degree of central condensation.

The theory of rotational fission shows that a star rotating as a rigid body can only break into two detached masses if there is no pronounced central condensation. Such extreme degrees of central condensation as those just described put fission quite out of the question; stars such as we have been considering would merely scatter themselves by a process of equatorial break-up. Minor adjustments, such as taking account of variations of molecular weight and inequalities in the rate of generation of energy, may possibly do something in the way of reducing the degree of central condensation, but it seems inconceivable that they should so far reduce it as to make rotational fission a possibility so long as the star rotates as a rigid body.

8. When a star is scattering its substance in the equatorial plane by equatorial break up, the fastest moving molecules or particles are

* *M.N.*, **85** (1925), p. 203.
† *Gas Kugeln*, p. 80.

thrown off first, and these carry an undue amount of angular momentum with them, and so unduly lessen the angular momentum of the equatorial layers which are left behind. Thus equatorial break up tends to set up inequalities in angular velocity, and as it proceeds the inner parts of a star will continually gain in speed of rotation over the outer parts.

It seems possible that finally the rapidly rotating inner parts, in which the variations in density are not very great, may behave much as the uniformly rotating mass does in the established theory, assuming in turn the shapes of Maclaurin spheroid, Jacobian ellipsoid, etc., and ending by fission into two distinct masses. During this process the outer part of the star merely acts as an atmosphere. Rotating at a different rate, its main function is to maintain pressure on the boundary of the inner part and so keep this inner part at comparatively uniform density throughout. The final result would appear to be a binary star of density nearer to that of the central regions of the parent star than to its mean density; this would accord with the general tendency to rather high densities exhibited by newly formed binaries. But the whole theory needs re-examination and probably recasting.

9. In conclusion let us glance at the corresponding problems for nebulæ. The value of ρ in a large spiral is about 10^{-21} times its value for the Sun, while T is probably a few hundred degrees at most, or say 10^{-5} times the central temperature of the Sun. With these very rough figures formula (8) would assign to κ a value equal to $10^{4.5}$ times that for the Sun or, say, 100,000. We cannot feel much confidence in this actual figure; apart from the extreme uncertainty of the numerical data it is suitable for soft X-radiation, although it is most likely that nebular radiation should be regarded mainly as either heat radiation or the very hard radiation formed by the annihilation of matter or as a mixture of both types. In any case κ must be large as a consequence of the low density of the nebula permitting a long free path to radiation.

If we take κ to be of the order of 10^5 and ρr^2 to be of the order of 10^{19}, the time necessary to reduce inequalities of angular velocity to half is found to be of the order of only 10^{14} seconds or 3 million years. This is very small in comparison with the total life of the nebula, being only about the time of a single rotation of the nucleus. Thus the nucleus ought to rotate with almost perfectly uniform velocity. This is in accord with spectroscopic measurements as well as with the circumstance that the observed shapes of the nuclei agree with those of masses of gas rotating as rigid bodies.

The question of rotation of the arms is more difficult. If these are orbits of masses of gas shot off from the periphery of the nucleus, their age can only be one or two revolutions of the nucleus or a few millions of years. If we can imagine there to be a continuous bridge of matter between the nucleus and the arms, then the arms ought in this period to have partially acquired the same angular velocity as the nucleus. In the extreme case of κ being so large that the period of equalisation is small compared with a rotation of the nucleus, the arms would be dragged on to rotate at the same rate as the nucleus. I

have shown * that in this extreme case the orbits, as shown by the shape of the arms, would be equiangular spirals, which is the curve they are actually observed to form. But the question needs much further critical discussion before this explanation of the equiangular curve of the spiral arms can claim acceptance.

10. The foregoing figures refer only to large spirals of the magnitude of M31 and M33. For smaller spirals the conditions may be intermediate between those just discussed and those prevailing in giant stars. Thus the period of equalisation of angular velocity may be intermediate between the 10^6 years found for the former and the 10^{12} years found for the latter. It will, at least, be longer than the period of rotation, so that the nebular arms ought not to rotate uniformly with the nebula, and possibly the nucleus ought not to rotate as a rigid body. In the latter case the phenomenon described in § 8 may occur; we may find the innermost regions of the nebula rotating faster than the outer and forming Jacobian ellipsoids, pear-shaped figures, and finally twin nebulæ, the whole being surrounded by the outer layers of the nebula rotating at a slower rate. It may be possible on these lines to obtain an explanation both of observed binuclear nebulæ and also of those nebular formations which suggest a Jacobian ellipsoid imprisoned in a Maclaurin spheroid—for instance, the so-called ϕ-nebulæ, the Z-shaped planetary nebulæ, and such planetary nebulæ as N.G.C. 7009. Exact analysis is difficult as the mere fact of the interior of the nebulæ being visible shows that there cannot be ordinary radiative equilibrium.

Note on the Problem of Cepheid Variation.

By Dr. P. ten Bruggencate.

1. In a recent paper,† Dr. Jeans has developed a new and very interesting theory on Cepheid variation. His suggestion is, that the oscillation of a Cepheid (and long-period) variable takes place about a state which is "ordinarily" stable but "secularly" unstable. The theory of the configurations of equilibrium of a rotating liquid shows that the pear-shaped figures are ordinarily stable but secularly unstable. The assumption of a pear-shaped figure for a Cepheid variable leads to a very satisfactory explanation of the light-curves with secondary maxima and of the period-luminosity law for stars brighter than absolute magnitude −2. The theory also gives plausible values for the density of Cepheids with a period of 6^d or longer. It is, however, noteworthy that there exist the following two important observational facts, wherein Jeans' theory fails.

2. It is well known that the amplitude of the secondary maxima of the light-curves of Cepheids decrease with decreasing period of light-variation. There exists no short-period Cepheid variable ($P < 1^d$) with

* *M.N.*, **77** (1917), p. 200.
† *Ibid.*, **85**, 797.

a secondary maximum in the light-curve; and, on the other hand, there exists no Cepheid variable of relatively long period (say $P > 15^d$), the light-curve of which possesses no secondary maximum. I cannot see how Jeans' theory in its present form can explain this fact. The pear-shaped figures being unsymmetrical, there will always arise secondary maxima in the light-curves, due to the rotation-term in Jeans' equation of light-variation. Concerning the existing relation between period and spectral type, there is also no explanation by Jeans' theory. But I hope that the following remarks will show how it can, at least qualitatively, be made more complete.

3. Jeans' model of a Cepheid variable—a homogeneous, incompressible, rotating liquid—is only a very rough approximation to reality. A further approximation would be reached by considering the figures of equilibrium of a rotating and radiating star. But since Jeans has shown * that the treatment of this problem by v. Zeipel fails for any actual star, there does not exist much hope of solving this problem. Nevertheless, I think that some qualitative results may be obtained in the following way:—

Eddington's model star, a star of the polytropic class $n=3$, gives, as we know, a satisfactory approximation to the actual "normal" star, that is, a star which never comes into a critical state. We must therefore study the figures of equilibrium of a rotating and radiating gaseous star of the polytropic class $n=3$. The configurations of equilibrium are determined by the equation

$$d(p+p_s)=\rho d\Phi,$$

where p is the gas pressure, p_s the radiation pressure, ρ the density, and Φ the gravitational (plus centrifugal) potential. Let us consider the two limiting cases (*a*) $p_s \ll p$, and (*b*) $p_s \gg p$.

(*a*) This case is realised in stars of small mass. The configurations of equilibrium of such a radiating mass will be, to a first approximation, the same as without radiation. Jeans has shown † that all polytropic models with $n > \frac{5}{6}$ break up equatorially. *Thus Eddington's model of a star with small mass will*, providing the density is small enough, *break up equatorially.*

(*b*) $p_s \gg p$ is realised in stars of very large mass. This case has been treated by Milne,‡ who was led to the result: "In a sufficiently massive rotating star in which the energy is evolved in accordance with Eddington's assumptions, the density is constant throughout the material. . . . Therefore . . . it must take up one of the forms of a rotating gravitating liquid." *Thus the mass breaks up by fission.*

In the case of equatorial instability there are no oscillations about an ordinarily stable, but secularly instable state. Pulsations arise only by fissional break-up.

I believe the main difficulty lies in deciding to what extent the limiting mathematical cases have a physical significance; that is, to

* *M.N.*, **85**, 933.
† *Problems of Cosmogony and Stellar Dynamics*, p. 186.
‡ *M.N.*, **83**, 140.

determine which masses lead to a purely equatorial, and which to a purely fissional break-up. Since both the limiting cases are certainly correct, there remains only the question of determining for which masses both kinds of instability may occur simultaneously.

4. I think that in the case of Cepheid variables (of short and long periods), where we have to assume masses of a considerable range (according to their distribution in star clusters), the above-mentioned two kinds of instability may be of great importance. In an actual Cepheid variable we shall have both kinds of instability simultaneously; but the smaller the mass—that is, the shorter the period—the more predominant will be the equatorial break-up. Thus it will show a more symmetrical figure and loses, the shorter the period is, the more mass by breaking up. On the other hand, we shall have a more predominant fissional break-up for a star of larger mass, that is, for a longer period of light-variation. Thus we shall have a more unsymmetrical figure, which loses the less mass the longer the period. From the behaviour of our radiating and rotating model in the two limiting cases which we have considered, we may conclude that *the character of breaking up is a function of the mass of the star.* It is easy to see that the displacement of the character of instability with decreasing mass from fissional break-up to equatorial break-up explains, at least qualitatively, the above-mentioned points in which Jeans' theory fails:

The more symmetrical figures, belonging to smaller masses, that is, to Cepheids with shorter periods, suppress the secondary maxima in the light-curves of the Cepheids with decreasing period. The increasing loss of mass by equatorial break-up for Cepheids of decreasing period results in advancing the star to an "earlier" spectral type.

5. Finally, I should like to call attention to the fact that, by the same considerations, the following statement of Henroteau * is easily explained: "... these short-period variables exhibit definite peculiarities ... which appear to definitely indicate the existence of a satellite, and which cannot be satisfactorily explained on the assumption of simple pulsation alone. ... Are we then to assume that in the short-period variables, and in them alone, there is present a satellite whose tidal action is superimposed upon the true Cepheid variation of the primary? Or is it more logical to suppose that the satellite is present in all cases, and forms one of the necessary conditions and causes of Cepheid variation, the secondary perturbations in the case of the longer-period stars being so small as in most cases to have escaped notice or be practically evanescent?" The matter ejected by equatorial break-up plays the rôle of the satellite which produces tidal action upon the primary.

In all these considerations we do not assume any new hypotheses, all conclusions being drawn immediately from the behaviour of the limiting cases (*a*) and (*b*), which can be treated mathematically.

Universitäts-Sternwarte, Göttingen:
1926 January 23.

* *Publ. of the Dom. Obs. Ottawa*, 2, No. 1.

Note on the Secular Motion of the Pole. By H. Spencer Jones, M.A., Sc.D., H.M. Astronomer at the Cape.

During recent years, from 1919 onwards, the motion of the Pole, as derived from the observations made at the International Latitude Variation Stations, has shown a secular displacement in addition to the usual annual and fourteen-monthly periodic displacements. The secular motion has been mainly in the x-co-ordinate of the displacements and corresponds to a displacement of the Pole, in the direction of Greenwich, by an amount approaching one-tenth of a second of arc. Since 1922·7 the international latitude observations have been under the control of Professor Kimura, and a new star-programme and new values for the mean latitudes of the observing stations have been adopted. The new values adopted for the mean latitudes show the following differences from the values adopted in the former programme * :—

	Latitude (Old–New).
Mizusawa	$-0''.25$
Carloforte	$-0''.07$
Ukiah	$-0''.07$

The change in the star-programme might produce a common mean displacement at the three stations. The large change in the latitude of Mizusawa is suggestive, however, of local crustal movements.

An apparent secular displacement of the Pole would result from the use of values of the proper-motions subject to a systematic error. The observations made at the international latitude stations during the earlier period 1900–1917 were discussed by Lambert,† who found a general tendency to a progressive increase in the latitudes of all the stations during this period. This was explained as due in part to cumulative errors in the declinations and in part to a progressive displacement of the Pole. Schlesinger ‡ showed that if Boss's proper-motions had been used in the international reductions, instead of Cohn's values, which were based on Auwers' system, the progressive increases in latitude at the several stations would have disappeared to a large extent, except in the case of Mizusawa.

The international observations during the period 1918–1922, in which the secular displacement was very marked, were discussed by Wanach.§ During this period only three stations were in operation. By neglecting the z-term, it is possible to eliminate entirely any errors in the declinations of the stars in the derivation of the components x and y of the variation. The x values so derived exhibit the same

* H. Kimura, *Jap. Journ. Astronomy and Geophysics*, 2, No. 3.

† "An Investigation of the Latitude of Ukiah, Cal., and of the Motions of the Pole," *U.S. Coast and Geodetic Survey, Special Publications*, No. 80, 1922; also *A.J.*, No. 804, 1922.

‡ *A.J.*, No. 798, 1922.

§ *Astr. Nach.*, No. 5314, 1924.

secular change as those deduced by the normal method, in which account is taken of the z-term. The large increase in x of recent years would appear, therefore, not to be due to systematic errors in the adopted proper-motions.

On the other hand, in the recent report of the Dudley Observatory, B. Boss* states that "the observed progressive changes in mean latitude at the international latitude stations are shown to be due to systematic errors in the proper-motions used and to a slow motion of the Pole." The period to which this refers is not stated.

The observations at Greenwich with the Cookson floating telescope show no tendency towards a secular displacement up to the end of 1922, but during 1923 and 1924 a displacement is apparent.† This is in the opposite direction to that shown by the international results, and may be due to systematic errors in the adopted proper-motions.

In view of the discontinuity during 1922 in the programme of the international observations and of the uncertainty whether the secular displacement shown by the results is a real or an apparent phenomenon, the observations of stars observed both above and below Pole at the Cape during the programme of meridian observations recently concluded have been examined. If the secular displacement is real and applicable to observations at the Cape, it should be indicated in the yearly mean corrections to the adopted latitude.

The declinations in the ledgers are derived using an assumed latitude $-33^\circ\ 56'\ 2''{\cdot}50$ and Pulkowa refractions. Corrections for instrumental flexure have not been applied. Preliminary corrections for latitude variation were applied which represented the periodic portion of the international values as published by Wanach and Kimura, the secular displacement having been abstracted. The values adopted for the latter were as follows :—

1918·0	+·01
19·0	+·03
20·0	+·05
21·0	+·07
22·0	+·09

The values after 1922·7 were not affected by secular displacement, owing to the adoption by Professor Kimura of revised values for the mean latitudes of the stations. These were fitted on to the earlier results freed from the secular change.

The values of $\Delta\delta$ (above Pole *minus* below Pole) are given in the following table for the stars used as azimuth stars, in the two positions of the instrument (IE and IW) respectively, and for stars other than azimuth stars, observed both above and below Pole, the observations in the two positions of the instrument being combined. For these latter stars the observations were not sufficiently numerous, in general, for the separate results in the two positions to be of value. The weights are indicated by the suffixes.

* *Popular Astronomy*, **33**, 668, 1925.

† *M.N.R.A.S., Geophysical Supplement*, **1**, 100, 148, 219.

Values of $\Delta\delta$ (*above Pole* minus *below Pole*).

Year.	Azimuth Stars. IE.	Azimuth Stars. IW.	Other Stars.	All combined.
1918	$+1''\cdot07_{41}$	$+0''\cdot66_{47}$	$+1''\cdot08_{15}$	$+0''\cdot88_{103}$
19	$+0\cdot70_{84}$	$+0\cdot59_{69}$	$+0\cdot94_{19}$	$+0\cdot69_{172}$
20	$+0\cdot81_{62}$	$+0\cdot72_{61}$	$+0\cdot61_{28}$	$+0\cdot74_{151}$
21	$+0\cdot94_{95}$	$+1\cdot10_{85}$	$+0\cdot98_{54}$	$+1\cdot00_{234}$
22	$+0\cdot50_{89}$	$+0\cdot70_{80}$	$+0\cdot58_{87}$	$+0\cdot59_{256}$
23	$+0\cdot44_{57}$	$+0\cdot47_{74}$	$+0\cdot45_{96}$	$+0\cdot46_{227}$
24	$+0\cdot35_{46}$	$+0\cdot27_{38}$	$+0\cdot35_{273}$	$+0\cdot34_{357}$

The first three series of figures show the same general features. There is a fairly progressive decrease in the values of $\Delta\delta$ shown in all the three, the year 1921 standing out as an exception, and the decrease is more rapid and more regular in the second half of the period than in the first half. The general agreement of the three series indicates that the variations from year to year are a real phenomenon and not merely a result of accidental errors of observation.

We may represent the values of $\Delta\delta$ as a first approximation by the formula

$$\Delta\delta = a + bt.$$

Although the assumption of a linear progression in the values of $\Delta\delta$ throughout the period is not strictly justifiable, the period is not sufficiently long to assume a more complicated representation. The inclusion of the term in t certainly secures a much better general agreement with observation than is possible without it.

If the unit of time for t is one year and t is measured from 1921·5, the following values for a and b are derived by least-squares solutions :—

	Azimuth Stars. IE.	Azimuth Stars. IW.	Other Stars.
a	$+0''\cdot72$	$+0''\cdot68$	$+0''\cdot73$
b	$-0\cdot110$	$-0\cdot042$	$-0\cdot124$

The agreement in the a values is satisfactory. The values of b are rather irregular, though all of the same sign, but are subject to a relatively large probable error.

Combining together the three series, as in the last column of the table above, a solution by least squares gives

$$\Delta\delta = +0''\cdot68 \pm 0''\cdot048 - (0''\cdot092 \pm 0''\cdot023)t.$$

Hence

$$\Delta\phi = \tfrac{1}{2}\Delta\delta \text{ (below–above)}$$
$$= -0''\cdot34 \pm 0''\cdot024 + (0''\cdot046 \pm 0''\cdot012)t.$$

The constant term, after correction for flexure, corresponds to a latitude

$$-33^\circ\ 56'\ 2''\cdot59.$$

The value derived in the last Cape Fundamental Catalogue was

$$-33^\circ\ 56'\ 2''{\cdot}51.$$

The term in t, representing the secular motion of the Pole, is in the same direction as the motion indicated by the international latitude observations. For, in the notation of the latter reductions,

$$\Delta\phi = x\cos\lambda + y\sin\lambda,$$

and, since the variation at the Cape is attributable mainly to the x term, the observed motion would correspond to an increase in x, as found in the international observations. The variation found at the Cape is somewhat larger than that indicated by the international results; two factors, however, need to be borne in mind in making the comparison. The comparison with the international figures is concerned mainly with the values of the latter up to 1922·7, but the progression in the Cape figures is most rapid towards the end, and a comparison up to 1922·7 would show better agreement. Thus if the year 1924 is excluded, the coefficient of t is reduced from 0″·046 to 0″·033. Secondly, the coefficient of t cannot be derived with great weight from a short period of observation, and the probable error of the derived coefficient is relatively large. The difference between the secular variation derived from the international observations and that resulting from the Cape meridian observations does not exceed twice the probable error of the latter determination.

We may therefore conclude that the Cape circumpolar observations during the period 1918–24 are in agreement with the international latitude observations in indicating a secular but non-uniform displacement of the North Pole of the Earth in the direction towards Greenwich. The present investigation is not affected by errors in the declinations or proper-motions of the stars, and Wanach's conclusion that the displacement derived form the international latitude observations could not be attributed to such errors is confirmed.

Preliminary Values of the Variation of Latitude at Greenwich in 1925.

(*Communicated by the Astronomer Royal.*)

The present paper contains the preliminary values of the latitude variation derived from observations made at the Royal Observatory, Greenwich, with the Cookson Floating Zenith Telescope during 1925. A fairly continuous series of observations was secured, the number of plates obtained being slightly above the average.

The reductions have been carried out in the same manner as in previous years.

The following table gives the values of the latitude variation, in

the sense mean latitude *minus* observed, as derived from the observations :—

Date.	Variation.	Number of Plates.
1925·019	−″·054	20
·103	−·071	10
·184	+·013	24
·266	+·069	18
·371	+·029	17½
·442	+·093	29½
·490	+·059	2½
·571	+·041	18½
·668	+·049	28
·773	+·010	16
·834	+·012	9½
·913	+·038	35½
1926·026	+·066	27

From a smooth curve drawn to represent the observed results as closely as possible, the following corrections, applicable to observed north polar distances, were derived :—

1925·0	−″·035	1925·6	+″·050
25·1	−·050	25·7	+·031
25·2	+·020	25·8	+·025
25·3	+·065	25·9	+·032
25·4	+·075	1926·0	+·050
25·5	+·070		

The results for 1922, 1923, and 1924 have been published in *Geophysical Supplement*, vol. i., No. 3, p. 102, No. 4, p. 148, No. 6, p. 219. Earlier results have been published in the *Monthly Notices*. The year 1925 completes the second 7-year period during which observations have been made at Greenwich with the Cookson Floating Telescope, and a final reduction of the observations will now be undertaken.

Faint Stars in the Selected Areas with large Proper Motion. First List from the Radcliffe Observatory, Oxford. By H. Knox-Shaw, M.A., Radcliffe Observer.

It will be some years yet before the programme of work on which the Radcliffe Observatory is engaged, the determination of the proper motions of stars in the Selected Areas, is completed, and in general the publication of results will be deferred until the whole, or at any rate a large part, of the work is finished. But it seems desirable to publish periodically the results for stars whose proper motions are

found to be large. The lower limit for this purpose has been taken as 0″·1 per annum.

Included in the programme are the Selected Areas Nos. 1–115 and twenty areas of the Special Plan, 135 areas in all. Two series of plates are being taken of each area at two epochs differing by not less than ten years. The first series consists of plates containing two exposures of 30 minutes made at the earlier epoch, which have been stored undeveloped and are being given two similar exposures at the later epoch. The second series consists of pairs of plates, the first being exposed at the earlier epoch in a similar way to a plate of the first series and then developed, while the second is exposed at the later epoch with its glass side turned towards the objective. Thus there are in general available for each star two values of its proper motion, which are to a large extent independent. The discordances between these two values provide a measure of the probable error of the proper motions derived for the stars in any given area. For those so far reduced the mean values are :

Probable Error of a Proper Motion in R.A.=±0″·0055,
Probable Error of a Proper Motion in Dec.=±0″·0058.

The work on ten areas has been completed, and in them thirteen stars have been found with proper motions exceeding 0″·1 per annum. Details are given in the table, most of the columns of which are self-explanatory. All the magnitudes given in the table are on the scale

Faint Stars in the Selected Areas with large Proper Motion.

Selected Area No.	Area measured.	No. of Stars measured.	No. of Comparison Stars.	Mean Mag. of Comparison Stars.	Limiting Magnitude.	Star No.	Magnitude.	R.A. 1900·0.	Dec. 1900·0.	Annual Proper Motion. μ_α.	μ_δ.	μ.
								h m s	° ′	″	″	″
30	60′×60′	143	80	13·3	15·2	399	13·6	10 35 13	+45 33·4	−0·090	−0·112	0·144
34	60×60	129	60	13·0	14·6	242	11·0	14 47 14	+44 51·1	+ ·035	− ·216	·219
						467	8·0	14 47 51	+45 20·5	+ ·006	− ·124	·124
36	50×50	166	88	13·1	14·8	—						
59	60×60	144	82	13·5	15·0	210	13·7	15 2 58	+29 35·8	− ·299	+ ·035	·301
						354	13·7	15 0 8	+29 52·4	+ ·238	− ·171	·293
60	60×60	149	83	13·0	14·7	260	8·0	15 55 3	+29 43·1	+ ·060	− ·089	·107
						275	11·8	15 56 32	+29 49·2	− ·168	− ·016	·169
						604	12·1	15 58 12	+30 21·9	+ ·064	− ·077	·100
61	50×50	201	106	13·2	14·9	—						
63	40×40	374	180	12·7	14·7	—						
91	60×60	128	64	12·7	14·4	578	13·0	23 14 20	+14 51·7	+ ·058	− ·130	·142
04	60×60	131	70	12·8	14·6	133	12·7	12 36 34	− 0 24·0	+ ·005	− ·122	·122
						460	13·2	12 37 56	+ 0 4·5	− ·102	+ ·034	·108
10	50×50	79	49	12·2	14·0	246	12·8	18 36 44	+ 0 0·4	− ·072	− ·165	·180
						409	11·2	18 36 29	+ 0 15·3	−0·059	−0·157	0·168

of the Durchmusterung of Selected Areas,* which is known to require in general positive corrections of a few tenths of a magnitude to reduce it to the international scale.† The magnitudes of the individual stars are those given in the Durchmusterung. The limiting magnitudes are not those of the faintest star measured in each area, but are the mean magnitudes of the faintest of the groups into which the stars were classed by the measurer. The proper motions are relative, and the mean magnitudes of the comparison stars to which they refer are given in the table. The positions of the stars have been taken from the Durchmusterung. The proper motions are the means of the values derived from the two series of plates.

The plates were measured by Mr. H. G. S. Barrett, who also supervised their reduction.

Diameters of the Sun Observed at Greenwich, 1915–1925.

By R. T. Cullen.

(*Communicated by the Astronomer Royal.*)

1. As the method and conditions under which the observations are made may have some bearing on the points raised later in the paper, it is perhaps necessary to give at once a few details.

The observations of the Sun made with the transit circle since 1915 are tangential pointings of a single wire on the Sun's edge or limb, the field of view showing a black wire against the Sun's disc screened by an eye-piece with green glass and slide giving three depths of intensity.

The horizontal diameter is observed by maintaining continuous tangentiality on both the first and second limbs over the same part of the field, as far as possible, *i.e.* about 8 seconds either side of the meridian. The quantity really determined as read from the chronograph is the sidereal time taken by the Sun's diameter crossing the meridian.

The vertical diameter is also a tangential pointing by a horizontal wire on both the north and south limbs, but as a portion of the disc only is visible in the field of view, the telescope has to be moved and the microscopes read for each limb. The observations of both north and south limbs are made between those of the first and second limbs.

2. These observations are naturally affected by personality, but as the position of the Sun's centre mostly depends upon the mean of two limbs, these personalities in the main are eliminated. However, for various reasons a number of observations are obtained when only one limb in either co-ordinate is observed. In these cases a semidiameter is necessary to reduce the limb to the Sun's centre. The semidiameter applied is that peculiar to the observer. These are derived by comparing annually the observed diameters arranged for each observer with those given in the *Nautical Almanac*. Thus the position of the

* *Harvard Annals*, **101**.
† *Ap. J.*, **61**, p. 303.

Sun when only one limb has been observed depends on the *N.A.* semi-diameter corrected by the observer's personality.

For the regular observers who have made nearly all the observations over the period 1915 to 1925, the personalities are:

Corrections to Nautical Almanac Diameter.

Observer.	Horizontal.	No. of Obs.	Wt.	Vertical.	No. of Obs.	Wt.
Mr. Jackson	− ″·06	185	3	+1″·00	208	3
,, Witchell	− ·48	292	4	− ·20	339	5
,, Cullen	− ·60	228	3	− ·50	224	3
,, Acton	− ·81	287	4	− ·92	325	5
,, Jeffries	+ ·09	109	1	·00	107	1
,, Barton	− ·99	58	1	− ·72	66	1
,, Symms	−1·47	62	1	− ·06	70	1
Mean	− ·57	..	17	− ·26	..	19

Indicating that in the mean the vertical diameter is observed greater than the horizontal by ″·31. This is in agreement with previous determinations, also with those by other investigators.

3. Examination of the observed diameters in the year 1925, when arranged for the various observers, seemed to suggest in each case a variation with the time of year; thus it appeared worth while to collect the results over a series of years, and this paper is brought forward mainly to exhibit these results.

The observations were arranged annually for each observer in the form "*N.A.*—Observed," from which each observer's personality was derived. From these are obtained monthly sums and means of the residuals since 1915, after allowing for each observer's mean personality, the mean of which is 0″·40.

These give for each observer:

Horizontal Diameter, 1915–1925.

	Mr. Jackson.	Mr. Witchell.	Mr. Cullen.	Mr. Acton.	Mr. Jeffries.	Mr. Barton.	Mr. Symms.
	s	s	s	s	s	s	s
Jan.	+·043	+·028	+·058	+·033	+·043	+·040	+·020
Feb.	+·010	−·004	−·010	+·050	+·045	+·080	+·040
Mar.	+·003	+·026	+·073	−·007	+·009	−·028	+·003
Apr.	−·042	−·015	−·011	+·002	−·038	+·070	−·051
May	−·021	−·027	−·036	−·015	−·055	+·012	−·019
June	−·060	−·045	+·004	−·031	+·005	−·055	−·033
July	+·015	−·008	−·008	−·019	+·001	−·040	−·048
Aug.	−·014	−·009	−·034	−·018	+·092	−·005	+·074
Sept.	+·009	−·019	−·043	−·008	+·027	−·036	−·023
Oct.	+·006	+·012	−·004	+·003	−·049	−·075	−·005
Nov.	+·003	+·039	+·014	+·004	−·050	+·079	+·046
Dec.	+·050	+·007	+·054	+·021	−·017	+·012	+·018

Vertical Diameter, 1915–1925.

Jan.	−″·19	+″·38	+″·49	+″·70	+ ″·60	+ ″·41	+ ″·72
Feb.	+·20	+·13	−·19	+·29	+ ·53	+ ·80	− ·66
Mar.	+·55	+·26	−·08	−·07	− ·05	− ·20	+ ·28
Apr.	·36	+·01	+·21	−·21	−1·19	+ ·97	+ ·15
May	−·51	−·36	−·26	−·34	− ·41	− ·66	− ·09
June	−·34	−·15	−·38	−·37	+ ·49	− ·59	− ·68
July	+·33	−·10	+·13	−·11	− ·28	+ ·18	− ·29
Aug.	−·13	+·11	−·30	−·31	+ ·74	− ·53	+ ·43
Sept.	−·43	+·09	−·42	−·04	+ ·27	− ·19	+1·01
Oct.	+·35	−·13	−·15	−·20	− ·35	− ·12	− ·62
Nov.	+·70	+·32	+·75	+·33	− ·13	+1·23	+ ·21
Dec.	+·62	+·27	+·99	+·46	+1·15	·00	+ ·34

Corresponding to the number of observations :

	Mr. Jackson.	Mr. Witchell.	Mr Cullen.	Mr. Acton.	Mr. Jeffries.	Mr. Barton.	Mr. Symms.	Total.
Jan.	7	21	16	26	11	7	5	93
Feb.	10	15	12	21	10	4	4	76
Mar.	6	24	14	22	8	8	6	88
Apr.	11	25	23	19	12	1	7	98
May	16	35	22	35	14	8	9	139
June	19	32	22	25	11	4	3	116
July	27	32	12	25	13	7	8	124
Aug.	14	36	20	33	8	4	5	120
Sept.	27	15	19	24	10	7	3	105
Oct.	25	27	24	21	12	4	2	115
Nov.	16	22	18	26	6	7	5	100
Dec.	11	16	7	16	3	5	5	63

These numbers are for the horizontal diameter; those for the vertical are not given, but differ by only a few units from the above.

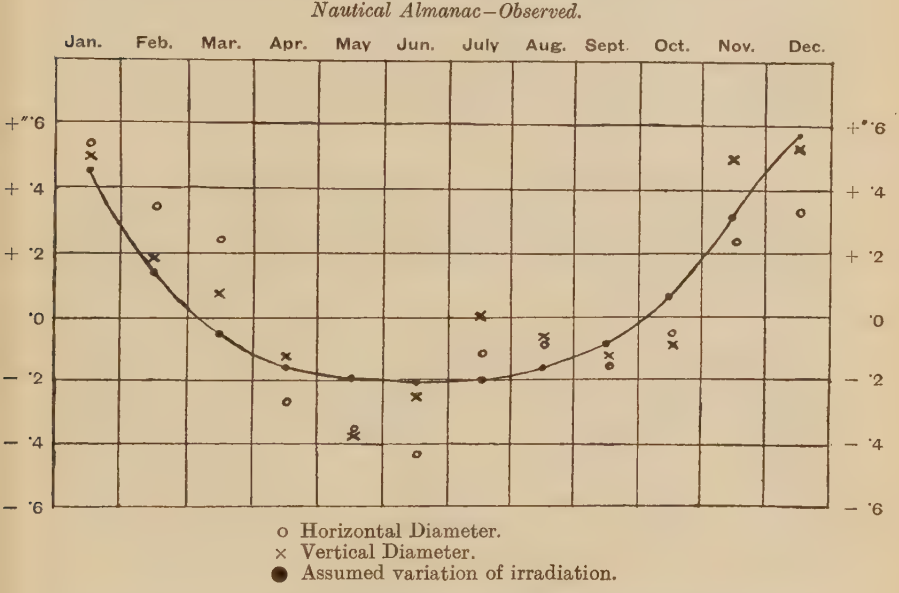
The mean monthly differences given by all observers, reduced to arc, are :

Month.	*N.A.*—Observed.	
	Horizontal.	Vertical.
	″	″
Jan.	+·53	+·49
Feb.	+·35	+·18
Mar.	+·24	+·08
Apr.	−·27	−·13
May	−·35	−·36
June	−·43	−·26
July	−·11	·00
Aug.	−·09	−·08
Sept.	−·15	−·12
Oct.	−·06	−·09
Nov.	+·24	+·49
Dec.	+·32	+·51

The mean discordance of one observation is ±1″·06 and ±″·91 for horizontal and vertical diameter respectively. Corresponding to a probable error of ±″·09 and ±″·08 for 100 observations, which, approximately, is the average number for each month in the 11 years.

The close accordance of the results from both co-ordinates is noteworthy, and points to something in the observations requiring explanation or investigation.

4. The most marked features are the variations with the time of year and the extreme range, amounting to one second of arc. The observed diameter is larger in summer than in winter.



To find some plausible explanation for these deviations, the following suggestions may be offered as presenting themselves most readily :—

(1) The theoretical diameter computed with an erroneous eccentricity.

(2) The limb observations being more disturbed in summer than in winter.

(3) The correction for irradiation, and its variation.

Dealing with each suggestion in turn, the first is at once deleted, because it would produce large errors in longitude. Any admissible error would not sensibly alter the diameter.

Secondly, the Sun's limb is likely to be more disturbed or to appear more wavy in summer, due to heat, than in winter. This effect on the observations is perhaps to increase the diameter in summer.

Thirdly, the effect of irradiation. For the Sun's diameter at the Earth's mean distance the *Nautical Almanac* adapts a constant of 1922″·36. This is an excess of 3″·10, for the effect of irradiation, over

Auwers's value of 1919″·26 used in the computation of eclipses. May the observed deviations be interpreted as a variation of this correction for irradiation ?

The conditions which would affect irradiation are :

(*a*) The different altitudes of the Sun, affecting atmospheric absorption.

(*b*) Variations in contrast between the Sun's disc and the background of the sky.

From these considerations it seems reasonable to assume that irradiation varies with the secant of the Sun's zenith distance, and on the previous page are plotted the observed deviations of horizontal and vertical diameters, together with a curve calculated by the formula $-''\cdot55+''\cdot30$ secant Sun's zenith distance, the constant term $-''\cdot55$ being introduced merely to prevent alteration of the mean diameter. A solution of the residuals in the form $x+y$ sec Sun's Z.D. actually gave

for horizontal diameter $-''\cdot55+''\cdot29$ sec Sun's Z.D.
for vertical diameter $-''\cdot56+''\cdot30$ sec Sun's Z.D.

If the *N.A.* diameter is altered to accord with the variations observed at Greenwich, the following monthly corrections are necessary :—

Month.	Sun's Dec.	Sun's Z.D.	Correction to *N.A.*	Month.	Sun's Dec.	Sun's Z.D.	Correction to *N.A.*
	°	°	″		°	°	″
Jan.	−21	72·5	−·45	July	+21	30·5	+·20
Feb.	−13	64·5	−·15	Aug.	+14	37·5	+·17
Mar.	− 2	53·5	+·05	Sept.	+ 3	48·5	+·10
Apr.	+10	41·5	+·15	Oct.	− 9	60·5	−·06
May	+19	32·5	+·19	Nov.	−18	69·5	−·31
June	+23	28·5	+·21	Dec.	−23	74·5	−·57

The results given in § 3, together with the above variations, imply corrections to the *N.A.* irradiation term of

(1) A constant correction of $-''\cdot4$.
(2) A variable correction of $+''\cdot55-''\cdot30$ sec Sun's Z.D.

To summarise, the

N.A. adopts $1919''\cdot26+3''\cdot10$,

and, accepting Auwers's constant as correct, the irradiation term $3''\cdot10$ needs a correction of

$+''\cdot15-''\cdot30$ sec Sun's Z.D.

5. Of the suggestions put forward, that of irradiation seems the most probable; but further investigation is required before definite conclusions may be drawn.

Further, these variations show that when one limb only of the Sun

has been observed, the position of the centre as deduced from the semidiameter, as at present applied, is liable to an error of $\pm''\cdot25$ at certain times of the year. As few observations are made on one limb only, and to these half-weight is given, the error is of no great importance.

Finally, an examination of the variation in the observed diameters synchronous with the sun-spot cycle yielded results that were inconclusive.

Royal Observatory, Greenwich :
1926 February 16.

Second Note on W. Herschel's Extended Nebulous Fields.

By J. G. Hagen, S.J., Assoc.R.A.S.

Since the first of these Notes was written Mrs. Isaac Roberts has kindly furnished us with positive contact prints of her late husband's original negatives, from which we have made enlarged negative pictures on bromide paper. The pictures measure two degrees in both dimensions, like the astrographic charts. They have no reticle impressed upon them, but in compensation contain a greater number of stars, a good deal beyond the visual limit of our 16-inch refractor.

1. Comparison with the sky shows that Roberts centered his plates very carefully on the positions given by Herschel. The $\Delta\alpha$ and $\Delta\delta$ of Table I. mark the differences between the guiding star and Herschel's place, in the sense (star—centre) expressed in minutes of arc, as in the first Note.

TABLE I.

H.'s No.	Centre (1925).	Guiding Star (1925).	Mag.	$\Delta\alpha$.	$\Delta\delta$.
	h m ° ′	h m ° ′		′	′
5	0 31·5+23 33	Berlin B 180 0 33·2+23 36	5·8	+22	+ 4
6	36·8+ 0 37	Nicolajew 116 38·2+ 0 39	8·1	+22	+ 2
7	39·4+41 18	Androm. Neb. 38·7+40 52	..	− 9	−26
8	40·9+39 24	Androm. Neb. 38·7+40 52	..	−27	+88
9	42·7+43 38	Bonn 666 46·1+44 36	6·9	+36	+58
10	0 50·0+43 43	Bonn 666 0 46·1+44 36	6·9	−41	+53
11	1 42·5+29 56	Cambridge E. 983 1 44·2+30 24	8·3	+23	+28
12	2 29·3+19 7	Berlin A 698 2 29·4+18 33	7·0	+ 1	−34

Table II. exhibits Herschel's description of what he saw, as published in the *Scient. Papers*, **2**, 462. The galactic latitude is added in order to show that Herschel's nebulosities are not restricted to regions near the galactic plane. His estimates of their extension along the Parallel and the Meridian are here counted from the centre of the field as in the first Note. From a comparison of the columns Parallel and Meridian it may be inferred that Herschel had more freedom of sweep in Decl. than in R.A.

TABLE II.

H.'s No.	Parallel.	Merid.	Description.	Gal. Lat.
	′	′		°
5	15	77	Much affected [with nebulosity]	−40
6	75	70	Appeared to be affected with very faint nebulosity	−63
7	47	90	Affected with nebulosity.	−22
8	39	85	Unequally affected with nebulosity.	−24
9	79	99	Suspected faint nebulosity.	−20
10	13	99	Suspected faint nebulosity.	−20
11	14	80	Suspected to be tinged with milky nebulosity.	−31
12	15	75	Much affected with nebulosity.	−37

2. Regarding our estimates of density, it may be remembered from the preceding Note that I. signifies the faintest nebular veils perceptible, and V. the densest clouds. All the observations were again confirmed by our Research Assistant, Dr. Fr. Becker.

Herschel No. 5.—The entire field is uniformly covered with clouds of density IV. and is poor in bright stars.

No. 6.—The general density is IV.–V., becoming V. in the *np* direction; field very poor in bright stars.

No. 7.—Herschel does not mention the Andromeda nebula, because his field is something else. What he calls "affected with nebulosity" is a thin cloudy veil varying in density from II. to III. in the northern half, becoming denser (III.–IV.) southward. Herschel saw this nebulosity extending over three degrees while he could not see the bright nebula larger than half this size.

The photograph of Roberts had an exposure of $3^h\ 3^m$, and shows in addition to the large nebula and N.G.C. 221 two other nebular spots, one *sp* and the other *np*, but no trace of Herschel's nebulosity.

No. 8.—This field is a continuation of No. 7, both forming one continuous nebulosity extending over four degrees. The centre assigned to No. 8 has density V., and the portion following it IV.–V. The common boundary of the two fields has density IV. A few bright stars in the *p* portion of the field.

No. 9.—General density III., becoming III.–IV. and IV. southward; several bright stars.

No. 10.—Almost uniform density III., tending towards II.–III. in the *nf* and to IV. in the *sf* corner of the field. Nos. 9 and 10 form a single cloudy field extending over four degrees in R.A. No. 10 is poorer in stars than No. 9, but has a bright double star in the *nf* corner.

Herschel No. 11.—Density throughout IV.–V., except the *nf* corner, where it is III. and IV.

No. 12.—The centre of the field has density V. over half a degree in extension; the rest varies between IV. and V., except in the *nf* corner of the field, where there is a thinner gap III.–IV. among brighter stars.

3. A few conclusions drawn from the above observations and some additional statements may serve to remove or prevent misconceptions.

(*a*) The main fact is this, that in the Herschel fields so far examined we in Rome see exactly what we have repeatedly referred to in our publications on Cosmic Clouds.

(*b*) The three fields 7, 9, and 10 show that Herschel had a keen eye for faint nebulosities. Density V. is always obvious at first sight, but here are densities III., which correspond to the average appearance of the sky and easily escape notice.

(*c*) Fields 7 and 8 form a double field, and the same holds for 9 and 10. A boundary where nebulosity changes into clear sky was not found in any of these fields, either by Herschel or by us, thus confirming our repeated statement that the heavens are largely covered with nebular clouds.

(*d*) Field No. 7 is sometimes erroneously quoted as having been photographed, due to a confusion of the Andromeda nebula with the Herschel field. Herschel could not qualify this brightest of all nebulæ as "affected with nebulosity," nor could he announce Sûfi's nebula as "not published before" (see *Scient. Papers*, **2**, 461).

It remains to be seen whether fields Nos. 22, 23, 24, 25, 27, 44, 46, which are also reported as photographed, are really so in a truer sense than is the case with No. 7.

Additional Note.

Regarding the remark appended to the first of these papers, we find no difference between Dr. Dreyer's view and our own, except for the unfortunate nomenclature *luminous* and *dark*, which we used because recommended to us by an expert of the highest authority. What we *see* cannot be entirely dark, whatever the source of its luminosity may be.

Evidently Herschel did not make any distinction between his first fields and the rest; nor do we. It was and is, on the contrary, our intention to show in a series of articles that all of Herschel's fifty-two regions present the same general aspect.

Can someone suggest an appropriate nomenclature for the nebular spots listed in the N.G.C. and the nebular clouds not contained therein? The lack of a definite nomenclature appears from Herschel's own words: "The nature of diffused nebulosity is such that we often see it joined to real nebulæ" (*Scient. Papers*, **2**, 464).

Vatican Observatory:
1926 *January.*

Occultations of Stars and Planets by the Moon, observed at the Astronomical Observatory, Kiew, in the Year 1925. By Professor S. D. Tscherny.

No.	Date 1925.	Greenwich Mean Time. h m s	Star.	Mag.	Dis. or Reap.	Telescope and Power.	Observer.
1	Apr. 4	18 14 49·67	ψ Leonis	5·6	D	R 145	T
2	4	18 14 49·66	ψ Leonis	5·6	D	G 58	K
3	4	18 14 49·48	ψ Leonis	5·6	D	B 61	L
4	4	19 34 19·91	ψ Leonis	5·6	R	R 145	T
5	27	17 49 36·84	64 Orionis	5·1	D	R 145	T
6	27	17 49 36·75	64 Orionis	5·1	D	K 24	D
7	27	18 55 39·62	64 Orionis	5·1	R	R 145	T
8	May 1	22 50 44·15	Neptune	7·7	I Contact	R 145	T
9	1	22 50 54·12	Neptune	7·7	II Contact	R 145	T
10	June 30	21 36 51·27	88 Virginis	6·5	D	R 90	T
11	Aug. 2	18 16 51·76	199 B Sagittarii	6·4	D	R 145	T
12	2	19 27 5·22	199 B Sagittarii	6·4	R	R 145	T
13	Sept. 23	16 24 59·12	ζ Ophiuchi	4·9	D	R 145	T
14	23	16 24 59·54	ζ Ophiuchi	4·9	D	B 61	L
15	23	16 24 59·65	ζ Ophiuchi	4·9	D	K 24	D
16	23	17 33 27·37	ζ Ophiuchi	4·9	R	R 145	T
17	26	17 43 16·44	50 Sagittarii	5·5	R	R 145	T
18	26	19 12 27·64	Anonyma	6·1	D	R 145	T
19	26	19 17 30·81	253 B Sagittarii	6·1	D	R 145	T
20	26	19 57 51·18	253 B Sagittarii	6·1	R	R 145	T

Instruments used :

R=243-mm. Merz-Repsold refractor.
K=120-mm. Utzschneider and Fraunhofer comet-seeker.
B=81-mm. Busch refractor.
G=70-mm. Goerz refractor.

Observers :

T=S. Tscherny.
D=M. Ditschenko.
L=E. Laurentjewa.
K=A. Karamischew.

Notes.

Nos. 5, 11, 13, 18, 19 : instantaneous.
Nos. 1, 3, 7, 9, 10, 14 : good.
Nos. 4, 8, 16, 17 : satisfactory.
Nos. 12, 20 : retardation.

Astronomical Observatory, Kiew,
1926 *January* 27.

Errata.

M.N., **83**, 363. For the period 143 the value of e should be -230, instead of -354.

M.N., **86**, 226. The first three lines of p. 226 refer to the comets at the foot of p. 225.

M.N., **86**, 241. Parallax of M 11, *for* 0″·008 *read* 0″·0008.